Interstellar scintillations of PSR B1919+21: space–ground interferometry

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Accepted 2017 March 9. Received 2017 March 6; in original form 2016 May 4

ABSTRACT
We carried out observations of pulsar PSR B1919+21 at 324 MHz to study the distribution of interstellar plasma in the direction of this pulsar. We used the RadioAstron (RA) space radio telescope, together with two ground telescopes: Westerbork (WB) and Green Bank (GB). The maximum baseline projection for the space–ground interferometer was about 60 000 km. We show that interstellar scintillation of this pulsar consists of two components: diffractive scintillations from inhomogeneities in a layer of turbulent plasma at a distance \( z_1 = 440 \) pc from the observer or homogeneously distributed scattering material to the pulsar; and weak scintillations from a screen located near the observer at \( z_2 = 0.14 \pm 0.05 \) pc. Furthermore, in the direction to the pulsar we detected a prism that deflects radiation, leading to a shift in observed source position. We show that the influence of the ionosphere can be ignored for the space–ground baseline. Analysis of the spatial coherence function for the space–ground baseline (RA–GB) yielded a scattering angle in the observer plane of \( \theta_{\text{scat}} = 0.7 \) mas. An analysis of the time–frequency correlation function for weak scintillations yielded an angle of refraction in the direction to the pulsar \( \theta_{\text{ref,0}} = 110 \) ms and a distance to the prism \( z_{\text{prism}} \leq 2 \) pc.

Key words: scattering – pulsars: general – pulsars: individual (B1919+21).

1 INTRODUCTION
Fluctuations of electron density in the interstellar plasma scatter radio waves from astronomical objects. An observer at the Earth detects a signal that is a convolution of the initial signal and a kernel that describes scattering in the interstellar plasma (Gwinn & Johnson 2011). Several effects are observed for pulsars corresponding to the scattering of the radio emission: intensity modulation in frequency and in time (scintillations), pulse broadening, angular broadening and signal dispersion in frequency.

A space radio telescope such as RadioAstron provides a great opportunity to measure the parameters of scattering. Separation of the effects of close and distant scattering material requires high spatial resolution. RadioAstron provides the space element of this interferometer for our observations. Technical and measured parameters of the RadioAstron mission have been described by Avdeev et al. (2012) and Kardashev et al. (2013) We observed several close pulsars in the Early Science program of RadioAstron (RAES), including pulsars B0950+08 and B1919+21. First results, published in the article of Smirnova et al. (2014), show that a layer of plasma located very close to the Earth, at 4.4–16.4 pc, is primarily responsible for scintillation of B0950+08. First indications that the nearest interstellar medium is responsible for the scintillations of pulsars B0950+08 and J0437−47 were discussed in earlier articles (Smirnova & Shishov 2008; Smirnova, Gwinn & Shishov 2006, see also Bhat et al. 2016). These pulsars have dispersion measures among the lowest observed, indicating a low column density of the plasma. A scattering medium located at a distance of about 10 pc from the Sun is also responsible for the variability of some quasars over periods of about an hour, when observed at centimetre wavelengths (Kedziora-Chudczer et al. 1997; Dennett-Thorpe & deBruyn 2002, 2003; Bignall et al. 2003, 2006; Jauncey et al. 2003). These observations of scattering of close pulsars and short-period variability of quasars indicate the existence of a nearby interstellar plasma component that has properties different from those of more distant plasma components.

The aim of the study reported here is to investigate the spatial distribution of the interstellar plasma towards the pulsar B1919+21. We show that two isolated layers of interstellar plasma lie in this
direction, one of which is localized at a distance of only 0.14 pc. Pulsar B1919+21 is a strong pulsar. Its period is $P = 1.3373$ s. It lies at Galactic latitude $3^\circ.5$ and longitude $55^\circ.8$. Its dispersion measure is $DM = 12.43$ pc cm$^{-3}$. The Cordes & Lazio (2002) model indicates that the pulsar distance is 1 kpc. Measurements of this pulsar’s proper motion yielded $\mu_\alpha = 17 \pm 4$ mas yr$^{-1}$ and $\mu_\delta = 32 \pm 6$ mas yr$^{-1}$ (Zou et al. 2005).

2 OBSERVATIONS

We conducted observations of PSR B1919+21 at an observing frequency of 324 MHz on 2012 July 4, using the RadioAstron 10-m space radio telescope together with the 110-m Green Bank (GBT) and $14 \times 25$-m Westerbork (WSRT) telescopes. Data were transferred from RadioAstron in real time to Puschino, where they were recorded using the RadioAstron Digital Recorder (RDR: Andrianov et al. 2014), developed at the Astro-Space Center of the Lebedeev Physical Institute (ASC). The Mark5B recording system was used for the ground telescopes. All telescopes recorded the frequency band from 316–332 MHz, with one-bit quantization for space telescope data and two-bit quantization for ground telescopes. Data were recorded for 4170 s, divided into scans of $421 P_1$ (563 s) and subintervals of $26 P_1$ (about 35 s). The primary data processing was done using the ASC correlator (Andrianov et al. 2014) with incoherent dedispersion. Data were correlated with 512 spectral channels in two selected windows: on-pulse and off-pulse; the width of each window was 40 ms (3 per cent of the pulsar period). An on-pulse window was centred on the maximum of the average profile and an off-pulse window was selected at half the pulsar period from the on-pulse window. The projected space–ground interferometer baseline was about 60 000 km.

3 DATA PROCESSING AND ANALYSIS

3.1 Dynamic spectrum and correlation functions

3.1.1 Dynamic spectrum

We formed complex cross-spectra between pairs of telescopes for all scans, in on- and off-pulse windows. In some cases, to increase the sensitivity, we averaged cross-spectra over four pulsar periods. To obtain dynamic spectra, we calculated the modulus of the cross-spectra and corrected for the receiver bandpass using the off-pulse spectra. To reduce the impact of broad-band intensity variations of the pulsar from pulse to pulse, we normalized each spectrum by its standard deviation, $\sigma(t)$. Fig. 1 shows the normalized dynamic spectrum of scintillation of pulsar B1919+21 for the Green Bank–Westerbork ground interferometer (GB–WB). We see clearly expressed large-scale sloping structures (slanting features), with scales of $\Delta f = 1–1.5$ MHz in frequency and $\Delta t \approx 1000$ s in time. Diffractive spots are strongly extended along the line $f = (\Delta f/\Delta t)t$. This drift indicates that refraction by a cosmic prism determines the structure of scintillation in the frequency–time domain. The regular dark bands along the frequency axis represent intervals when the signal was not recorded and were filled with values of zero. The narrower grey bands show the pulse-to-pulse variability intrinsic to the pulsar.

3.1.2 Drift and dual frequency scales

To determine the drift rate of the diffractive structure, we calculated the position of the maximum of the mean cross-correlation between spectra separated in time by lags $\kappa P_1$, where $\kappa = 1, 2, \ldots$ and $P_1$ is a pulse period in s. We found three separated bands with the slope $\Delta f/\Delta t = 1.5$ MHz/1000 s. We obtained this slope by a least-squares
A correlation analysis of the dynamic spectra provides the scales of

3.1.3 Determination of frequency scales

A correlation analysis of the dynamic spectra provides the scales of

fit to the positions of maxima. Fig. 2 shows spectra of several strong
pulses separated in time, with time increasing from bottom to top
in the figure.

Two scales of structure are visible in the spectra: small-scale structure with a frequency scale of about 400 kHz and large-scale structure with frequency scale of about 1500 kHz. These scales are the approximate full width at half-maximum amplitude of the features. At smaller separations in time (as in spectra ‘a’ and ‘b’ in the figure, separated by 11 s), the fine structure is the same. Over longer separations (as in spectra ‘b’ and ‘c’, separated by 200 s) the fine structure changes, but the large-scale structure retains its shape. The modulation index, defined as \( m(t) = \sigma(t)/\langle I_f(t) \rangle \), varies from 0.7–1.0 on a time-scale of the order of 500 s. This variation indicates that the statistics are not sufficient to determine the correct value of \( m(t) \). However, the fact that the modulation index is close to 1 confirms that the scattering is strong.

3.1.4 Determination of time-scale

Fig. 4 shows the average cross-correlation coefficient between pairs of spectra as a function of time for space–ground (upper) and ground (lower) baselines. The visible break in the slope of the ACF near a lag of \( \pm 300 \text{ kHz} \) for the ground baseline (GB–WB: Fig. 3, lower) indicates the presence of structure on two scales. No break appears in the ACF for the space–Earth baseline: its shape corresponds to the small-scale structure only. To determine the widths and relative amplitudes of these structures, we fit the ground baseline with a sum of exponential and Gaussian functions. We obtained frequency scales of \( \Delta f_{\text{dif}} = 330 \text{ kHz} \) and \( \Delta f_{\text{wide}} = 700 \text{ kHz} \) (half width at half-maximum amplitude), with amplitudes of 0.84 and 0.15 for the small- and large-scale structures, respectively. As will be shown below in Section 4, the small-scale structures arise from scattering of radiation in the distant layer of the turbulent medium as diffractive scintillation and the large-scale structures in the layer located close to the observer as weak scintillation.

3.2 Theoretical relations, ionospheric effects, correlation functions

Much of our analysis in this article follows that of Smirnova et al. (2014) in general outline, but here we describe new details connected with ionospheric effects and use different definitions for the correlation functions. Let \( h(f, t) \) be the spectrum of the field of initial pulsar emission in the absence of any turbulent medium, where \( f = v - v_0 \) is the offset of the observing frequency \( v \) from the band centre \( v_0 = 324 \text{ MHz} \) and \( t \) is time. This spectrum \( h(f, t) \) also includes instrumental modulation of emission in the passbands of the receivers. After propagation through the turbulent interstellar baseline, we calculated the ACF as the modulus of the average covariance function of the complex cross-spectra. The corresponding expressions are presented in Section 3.2. This procedure is required when the contribution of the noise is greater than or comparable with the signal level and when ionospheric effects are small, as was the case for our space–ground baseline. Otherwise, the contribution of noise will distort the ACF.

The dependence of the average cross-correlation coefficient between pairs of spectra spaced by the corresponding time interval \( \Delta t = 4P_k, k = 1, 2, \ldots \), for space–ground (upper) and ground (lower) baselines.
medium, the spectrum of the electric field for one antenna can be represented as
\[ E(\rho, f, t) = h(f, t)u(\rho, f, t) \exp[-iS(\rho, f, t)]. \]

where the modulation coefficient \( u(\rho, f, t) \) is determined by propagation through the interstellar medium and \( \rho \) is the spatial coordinate in the observer plane, perpendicular to the line of sight. The phase \( S(\rho, f, t) \) is determined by the ionosphere and cosmic prism. Multiplying \( E(\rho, f, t) \) by \( E^*(\rho + b, f, t) \) and averaging over the statistics of the source, we obtain the quasi-instantaneous response of an interferometer with a baseline \( b \), the cross-spectrum of the electric field:
\[ I(\rho, \rho + b, f, t) = E(\rho, f, t)E^*(\rho + b, f, t) = H(f, t)(\rho, \rho + b, f, t) \times \exp[-i\Delta S(\rho, f, t)], \]

where
\[ j(\rho, \rho + b, f, t) = u(\rho, f, t)u^*(\rho + b, f, t), \]
\[ H(f, t) = (h(f, t)h^*(f, t))_h. \]

The subscript \( h \) indicates averaging over the statistics of the noise-like electric field of the source. We assume that the intrinsic spectrum of the source, and of our instrumental response, is flat: \( H(f, t) = 1 \). The phase difference between the antennas at either end of the baseline \( \Delta S(\rho, f, t) \) consists of two components, from interstellar refraction and the ionosphere:
\[ \Delta S(\rho, b, f, t) = \Delta S_{\text{ion}}(\rho, b, f, t) + \Delta S_{\text{ref}}(b, f). \]

For a fixed baseline, the refractive component of the interferometer phase, \( \Delta S_{\text{ref}}(b, f) \) depends only on \( f \):
\[ \Delta S_{\text{ref}}(b, f) = 2\pi \left( \frac{f}{c} \right) b \cdot \Theta_{\text{ref},0}. \]

Here \( \Theta_{\text{ref},0} \) is the refractive angle at frequency \( v_0 \).

The ionospheric component can be represented as
\[ \Delta S_{\text{ion}}(\rho, b, f, t) = \Delta S_{\text{ion}}(\rho, b, f = 0, t) + \frac{f}{v_0} \Delta S_{\text{ion},0} \]
\[ + \frac{f}{v_0} (t - t_0) \Delta S_{\text{ion},1}, \]
\[ \Delta S_{\text{ion},0} = \Delta S_{\text{ion}}(\rho, b, f = 0, t = t_0), \]
\[ \Delta S_{\text{ion},1} = T \left. \frac{d}{dt} [\Delta S_{\text{ion}}(\rho, b, f = 0, t)] \right|_{t=t_0}, \]

where \( T \) is the time span of the observations and \( t_0 \) is the time at the middle.

Fig. 5 shows the values of the real and imaginary parts of the interferometer response for one selected frequency channel as a function of time: \( \text{Re}[I(\rho, \rho + b, f, t)] \) (upper) and \( \text{Im}[I(\rho, \rho + b, f, t)] \) (lower). In addition to the amplitude fluctuations corresponding to the dynamic spectrum, we see periodic fluctuations with a characteristic period of about 70 s, phase-shifted by 90°. Changes of the ionosphere in time cause these fluctuations. Therefore, to analyse the data from the ground interferometer we must work with the moduli of the cross-spectra. For the space–ground interferometer, the influence of the ionosphere was much smaller, as will be shown below, and so the data processing used the complex cross-spectra.

Multiplying \( I(\rho, \rho + b, f, t) \) by its complex conjugate at frequency \( f + \Delta f \) (where \( \Delta f = \text{frequency shift} \) and averaging over time and frequency, we obtain
\[ \langle I(\rho, \rho + b, f, t)I^*(\rho, \rho + b, f + \Delta f, t) \rangle = \langle (j(\rho, \rho + b, f, t))^2 (\rho, \rho + b, f + \Delta f, t) \rangle \phi(\Delta f) \]
\[ \times \exp[-i(\Delta f/v_0)]\Delta S_{\text{ion},0} + \Delta S_{\text{ref},0}, \]

where
\[ \phi(\Delta f) = \left\langle \exp \left[ -i \frac{\Delta f}{v_0} \left( t - t_0 \right) \Delta S_{\text{ion},1} \right] \right\rangle, \]

In equation (9), the average corresponds to an integration over \( t \) from \((t_0 - T/2)\) to \((t_0 + T/2)\). The phase difference at frequency \( v_0 \) is \( \Delta S_{\text{ref},0} \).

The modulus of the averaged correlation in frequency \( f \) of the interferometer response \( I \) is
\[ J_1(b, \Delta f) = \| I(\rho, \rho + b, f, t)I^*(\rho, \rho + b, f + \Delta f, t) \|. \]

In contrast, the averaged modulus of the correlation in frequency \( I \) is
\[ J_2(b, \Delta f) = \| I(\rho, \rho + b, f, t)I^*(\rho, \rho + b, f + \Delta f, t) \| \]

Fig. 3 (upper panel) shows the modulus of the averaged correlation function \( I \) for the space–ground interferometer, as defined in equation (10). The imaginary part of the second moment divided by its modulus is \( \phi(\Delta f) \).

Fig. 6 shows values of \( J_2(\Delta f) \) and \( \phi(\Delta f) \) for the ground interferometer and \( \phi(\Delta f) \) for the space–ground interferometer. We see that for the ground interferometer the ionospheric phase is larger and varies more rapidly with frequency than for the space–ground interferometer.

The value of \( \phi(\Delta f) \) is zero at \( \Delta f = 0 \), as defined by the factor \( \sin[-(\Delta f/v_0)\Delta S_{\text{ion},0} + \Delta S_{\text{ref},0}] \). However, the zeros of \( \phi(\Delta f) \) for the ground interferometer at \( \Delta f = \pm 3 \text{ MHz} \) are defined by \( \phi(\Delta f) \). If we set the argument of \( \phi \) in equation (9) equal to \( \pi \) at \( \Delta f = 3 \text{ MHz} \), we obtain \( \Delta S_{\text{ion},1} \approx 600 \text{ radians} \) for the ground interferometer and \( \Delta S_{\text{ion},1} \leq 200 \text{ radians} \) for the space–ground interferometer. From
Similarly, the structure function for the ground interferometer (GB–WB) as a function of $f$ (equation 11); (b) $\varphi_1(f)$, the ratio of the imaginary part of the correlation to its modulus for the ground interferometer (GB–WB) as a function of $f$; (c) the same ratio $\varphi_1(f)$ for the space–ground interferometer (RA–GB).

these values, it follows that the response of the ground interferometer is greatly distorted by the ionospheric phase, but the signal amplitude greatly exceeds the noise. Thus, we use equation (11) to calculate the correlation function of amplitude fluctuations for the ground baseline. For the space–ground interferometer, the situation is reversed: the noise exceeds the signal and we cannot use equation (11). However, we can neglect the phase distortion of the interferometer response and use equation (10) to determine the correlation functions of amplitude fluctuations on the space–ground baseline.

3.3 Structure functions of the interferometer response fluctuations

Structure functions provide insight into the correlation function, as discussed in earlier articles (Smirnova et al. 2014). For the space–ground interferometer on baseline $b_s$, we calculate the structure function in frequency difference $\Delta f$ and time difference $\Delta t$ using $J_1(b_s, \Delta f = 0, \Delta t = 0) - J_1(b_s, \Delta f, \Delta t)$. We normalize this expression by $J_1(b_s, \Delta f = 0, \Delta t = 0) - J_1(b_s, \Delta f = \Delta f^*, \Delta t = 0)$, where $\Delta f^* \gg \Delta f_{\text{dif}}$. Hence we obtain the structure function for the space–ground interferometer:

$$SF_s(b_s, \Delta f, \Delta t) = \frac{(J_1(b_s, \Delta f = 0, \Delta t = 0) - J_1(b_s, \Delta f, \Delta t))}{(J_1(b_s, \Delta f = 0, \Delta t = 0) - J_1(b_s, \Delta f^*, \Delta t = 0))}.$$  \hspace{1cm} (12)

Similarly, the structure function for the ground interferometer with baseline $b_g$ yields the normalized structure function

$$SF_g(b_g, \Delta f, \Delta t) = \frac{(J_2(b_g, \Delta f = 0, \Delta t = 0) - J_2(b_g, \Delta f, \Delta t))}{(J_2(b_g, \Delta f = 0, \Delta t = 0) - J_2(b_g, \Delta f^*, \Delta t = 0))}.$$  \hspace{1cm} (13)

Fig. 7 shows average time (upper) and frequency (lower) structure functions (SF) of intensity variations for the Green Bank–Westerbork (GB–WB) baseline, presented on a log–log scale. The arrow marks the frequency lag of the observed break in the structure function, 300 kHz.

Fig. 8 shows the average frequency structure functions for ground (GB–WB) and space–ground (RA–GB) baselines at zero time shift.

We performed linear fits to the logarithmic data for these two structure functions over the ranges $\Delta f_{\text{amp}} < \Delta f < \Delta f_{\text{dif}}$ and $\Delta f_{\text{amp}} < \Delta f < \Delta f_{\text{dif}}$, respectively, where $\Delta f_{\text{amp}}$ and $\Delta f_{\text{amp}}$ are the sampling intervals in time and frequency for our data. We obtained $\beta_f = 0.90 \pm 0.03$ for the frequency structure function and $\beta_t = 1.73 \pm 0.02$ for the time structure function. The resulting relation between frequency and time structure functions $\beta_f = \beta_t/2$ corresponds to a diffractive model for scintillation (Shishov et al. 2003).

The power-law index $n$ of the spectrum of density inhomogeneities responsible for scattering is connected with the index of $SF(t)$ through the relation $n = \beta_f + 2 = 3.73$ (Shishov et al. 2003).

Fig. 8 shows the average frequency structure functions for the ground (line) and space–ground (dashed line) baselines at zero time shift. Evidently the levels of the SF differ by about 0.2–0.3. This corresponds to the relative contributions of the two frequency scales in the scintillation spectra to the ground baseline, as seen in Figs 2 and 3 and discussed in Section 3.1.3 above. The ratio of their amplitudes is consistent with our fit to two components in the average frequency correlation function for the ground baseline. The space–ground baseline shows no such break in the structure function in Fig. 8, or what would be the corresponding structure in the correlation function shown in Fig. 3. Rather, the space–ground structure function displays only the narrower frequency-scale component.
where $D_s(\Delta \rho)$ is the spatial structure function of phase fluctuations:

\[ D_s(\Delta \rho) = \langle \phi(x + \Delta \rho) - \phi(x) \rangle_s, \tag{17} \]

where $\phi(x)$ is the screen phase at $x$. In the case of a spherical wave at the observer plane, we have

\[ D_s(\Delta \rho) = \int_0^z D(\Delta \rho) \, dz', \tag{18} \]

where $D$ is a gradient of $D_s(\Delta \rho)$ along the $z$-axis and

\[ \Delta \rho = \left( \frac{z - z'}{z} \right) b. \]

The integration is from the observer at $z' = 0$ to the pulsar at $z = z$.

For the space–ground interferometer, we used equation (15) with $b = b_g$, where $b_g$ is the interferometer baseline. According to equation (15), for $\Delta f = 0$ we have $J_1(b_s, \Delta f = 0) = 1 + |B_s(b_s)|^2$ and for $\Delta f > \Delta f_{\text{ref}}$ we have $J_1(b_s, \Delta f > \Delta f_{\text{ref}}) = |B_s(b_s)|^2$. As displayed in Fig. 3, we find $J_1(b_s, \Delta f = 0) = 1.4 \times 10^{-4}$ and $J_1(b_s, \Delta f > \Delta f_{\text{ref}}) = 2.4 \times 10^{-5}$ at $\Delta f = 2$ MHz. Thus,

\[ \frac{J_s(b_s, \Delta f > \Delta f_{\text{ref}})}{J_s(b_s, \Delta f = 0)} = \frac{|B_s(b_s)|^2}{1 + |B_s(b_s)|^2} = 0.17. \tag{19} \]

From this equation, we obtain

\[ |B_s(b_s)|^2 = 0.20. \tag{20} \]

From the analogous calculation for the ground interferometer (Fig. 6a), we obtain

\[ \frac{J_s(b_g, \Delta f > \Delta f_{\text{ref}})}{J_s(b_g, \Delta f = 0)} = \frac{|B_s(b_g)|^2}{1 + |B_s(b_g)|^2} = 0.50. \tag{21} \]

This implies

\[ |B_s(b_g)|^2 = 1. \tag{22} \]

This result implies that the ground interferometer does not resolve the scattering disc; mathematically, it means that $|b_g| \ll 1/k\theta_{\text{scat}}$.

4 MODEL OF THE TURBULENT INTERSTELLAR MEDIUM

Our analysis leads to the following model for the scintillation. The material responsible for scintillation of PSR B1919+21 consists of two components: strong diffractive scattering in a layer of turbulent density inhomogeneities at a distance of $z_1$, which is responsible for the small-scale structure in the spectra, and weak and refractive scattering in a layer of turbulent density inhomogeneities close to the observer at a distance of $z_2$, which is responsible for the large-scale structure. The spatial structure function of phase fluctuations $D_s(\Delta \rho)$, as described in Section 3.4 above, characterizes the two screens.

We describe here a physical model for the distribution of scattering material that explains our observations. Suppose we have a cosmic prism, located close to the observer at a distance of $z_{\text{prism}}$, which deflects the beam with an angle of refraction $\Delta \theta_{\text{ref}}$. Let $\theta_{\text{ref,0}}$ be the resulting shift of source position visible in the observer plane at frequency $v_0$. The difference in refraction angle at a nearby frequency $v_0 + f$ is (Shishov et al. 2003)

\[ \Delta \theta_{\text{ref}} = 2 \frac{f}{v_0} \theta_{\text{ref,0}}. \tag{23} \]

Suppose further that phase screen 2 is located close to the observer and that the distance between the observer and phase screen 2 is...
For saturated scintillations, the second moment \( J(\rho, \Delta f, \Delta t) \) consists of two components: diffractive and refractive (Prokhorov et al. 1975). The diffractive component can be represented as

\[
J(\rho, \Delta f, \Delta t)_{\text{dif}} = \langle j(\rho, \rho + b, f, t) \rangle_{\text{dif}}
\]

\[
= \langle j(\rho, \rho + b, f, t) \rangle (j^*(\rho, \rho + b, f, t))
\]

\[
+ B_j(\Delta f, \Delta t).
\]

(30)

Here \( B_j(\Delta f, \Delta t) \) is a frequency–time correlation function of flux fluctuations independent of the projected baseline:

\[
B_j(\Delta f, \Delta t) = \langle u(\rho, f, t) u^*(\rho, f + \Delta f, t + \Delta t) \rangle
\]

\[
\times (u(\rho + b, f, t) u^*(\rho + b, f + \Delta f, t + \Delta t))
\]

\[= |B_u(\Delta f, \Delta t)|^2. \]

(31)

Pulsar motion at a transverse speed \( V_p \) is primarily responsible for variations of flux density with time, producing scintillations from phase screen 1 at distance \( z_1 \). The pulsar motion leads to a shift of the beam in the plane of the phase screen:

\[
\Delta \rho_{s,t} = \frac{z_1}{z} V_p \Delta t.
\]

(32)

For \( \Delta f = 0 \), we have

\[
B_u(\Delta t) = \exp \left[ -\frac{1}{2} D_{S,1}(|\Delta \rho_{s,t}|) \right]
\]

\[= \exp \left[ -\frac{1}{2} \left( \frac{\Delta t}{\Delta t_{\text{dif}}} \right)^{\nu} \right], \]

(33)

with

\[
\Delta t_{\text{dif}} = \frac{z}{z_1 \kappa \theta_{\text{cat},1} |V_p|}.
\]

(34)

Correspondingly,

\[
B_j(\Delta t) = \exp \left[ -\frac{1}{2} D_{S,1}(|\Delta \rho_{s,t}|) \right] = \exp \left[ -\left( \frac{\Delta t}{\Delta t_{\text{dif}}} \right)^{\nu} \right].
\]

(35)

For a ground baseline, the projected length \( b_\parallel \) is much smaller that the coherence scale of the field \( 1/\kappa \theta_{\text{cat},1} \):

\[
b_\parallel \ll \frac{1}{\kappa \theta_{\text{cat},1}}
\]

(36)

and, correspondingly,

\[
B_u(\rho, \rho + b_\parallel) = 1.
\]

(37)

For a space–ground baseline, the projected length \( b_\parallel \) is larger than the coherence scale of the field:

\[
b_\parallel > \frac{1}{\kappa \theta_{\text{cat},1}}
\]

(38)

and, correspondingly,

\[
B_u(\rho, \rho + b_\parallel) = \exp \left[ -\frac{1}{2} \left( \frac{z - z_1}{z} \frac{b_\parallel}{\kappa \theta_{\text{cat},1}} \right)^{\nu} \right].
\]

(39)

Here, \([z - z_1]/z]\) is the sphericity factor, which converts the baseline \( b_\parallel \) into the distance between beams in the phase screen plane at distance \( z_1 \). Comparison of \( J(\rho, \Delta f = 0) \) and \( J(\rho, \Delta f > \Delta f_{\text{dif}}) \) allows us to estimate the spatial coherence function (interferometer visibility), as in Section 3.4 above.

5 RESULTS

As mentioned above, the time correlation function of interferometer response fluctuations is determined primarily by pulsar motion with

\[\text{Figure 10. Schematic illustration of model geometry, showing radiation at a lower frequency \( f \) (dark line) scattered by the strongly scattered screen 1 into a range of angles (dark shading), deflected by the cosmic prism and scattered further by the weakly scattering screen 2 before reaching the observer plane. The lighter path and shading shows a higher frequency \( f + \Delta f \).}\]
transverse velocity $V_p$. Pulsar motion causes the beam in the plane of phase screen 1 by $\Delta \rho_{1,1}$ (equation 32). The temporal correlation function of flux density shown in Fig. 4 yields $\Delta \tau_{\text{obs}} = 290 \text{ s}$. We fitted the model given by equation (35) to the shift of the structure function shown in Fig. 7 to find the index $\alpha_1$. The fit yielded $\alpha_1 = 1.73$. The normalized spatial correlation function of flux fluctuations for the space–ground baseline $b_s$ is (equation 20) $|b_s(b_s)|^2 = 0.20$.

From the projected length of the ground–space baseline $b_s = 6 \times 10^9 \text{ cm}$ we find $b_{\text{dif}}$:

$$b_{\text{dif}} = \frac{z}{z - z_1} k \theta_{\text{scat,1}} = 4.6 \times 10^6 \text{ cm}. \quad (40)$$

Using the measured proper motion of $\mu_0 = 17 \pm 4 \text{ mas yr}^{-1}$, $\mu_z = 32 \pm 6 \text{ mas yr}^{-1}$ (Zou et al. 2005) and an assumed distance to the pulsar $z = 1 \text{ kpc}$ (Cordes & Lazio 2002), we obtain a pulsar tangential velocity $V_p = 200 \text{ km s}^{-1}$. Comparison of $|V_p| \Delta \tau_{\text{dif}}$ with $b_{\text{dif}}$ yields $z_1/(z - z_1) = b_{\text{dif}}/(|V_p| \Delta \tau_{\text{dif}}) = 0.78$. Hence, $z_1 = 0.44 z = 440 \text{ pc}$. Therefore, the screen is located approximately halfway between the pulsar and the observer. From knowledge of $b_{\text{dif}}$ and $z_1$, we obtain $\theta_{\text{scat,1}} = 1.2 \text{ mas}$. In the observer plane, $\theta_{\text{obs}} = [(z - z_1)/z] \theta_{\text{scat,1}} = 0.7 \text{ mas}$.

The normalized frequency correlation function $R(\Delta f)$ of intensity variations is determined by the diffractive scintillations. Ostashev & Shishov (1977) found that $R(\Delta f)$ is

$$R(\Delta f) = 1 - \frac{1}{2} \left( \frac{\Delta f}{\Delta f_{\text{dif}}} \right)^{\alpha_1}, \quad \Delta f < 2 \Delta f_{\text{dif}},$$

$$\Delta f_{\text{dif}} = \frac{c}{4\pi A(\alpha_1) z (z_1/(z - z_1)) (\theta_{\text{obs,1}})^2}. \quad (41)$$

Taking $\alpha_1 = 1.73$, we find, for the constant $A(\alpha_1) = [2\Gamma(1 + \alpha_1/2) \cos\pi \alpha_1/4]^{2\alpha_1} \approx 0.34$, where $\Gamma$ is the complete gamma function. Using our estimated values for $z_1$ and $\theta_{\text{scat,1}}$, we find $\Delta f_{\text{dif}} = 290 \text{ kHz}$, which coincides very well with our measured value $\Delta f_{\text{dif}} = 330 \text{ kHz}$.

If we suppose the scattering material is homogeneously distributed between observer and pulsar, then the frequency diffraction scale will be determined by the relations

$$\Delta f_{\text{dif}} = \frac{c}{4\pi B(\alpha_1) z (\theta_{\text{obs,1}})^2}, \quad (43)$$

$$B(\alpha_1) = \left[ \frac{2 \Gamma(1 + \alpha_1/2)^3 \cos\pi \alpha_1/4 (1 + \alpha_1)}{\Gamma(2 + \alpha_1)} \right]^{2\alpha_1/\alpha_1} \approx 0.18. \quad (44)$$

We obtain $\Delta f_{\text{dif}} = 430 \text{ kHz}$, which also coincides with the observations. Thus, our measurement of $\Delta f_{\text{dif}}$ agrees with either a thin-screen model (located at $z_1 \approx 0.44 z$) or homogeneously distributed scattering material, to about the 30 per cent accuracy of our measurement. The consistency of the measured and calculated frequency scales suggests that the assumed pulsar distance of $z = 1 \text{ kpc}$ corresponds to the actual distance.

The spatial correlation function for weak scintillations from the inhomogeneities of layer $z_2$ can be represented as (Smirnova et al. 2014)

$$R(\Delta \rho_2) = D_{5,2}(\Delta \rho_{2,\text{Fr}}) - D_{5,2}(\Delta \rho_2) = D_{5,2}(\Delta \rho_{2,\text{Fr}}) \left[ 1 - \left( \frac{\Delta \rho_2}{\Delta \rho_{2,\text{Fr}}} \right)^{\alpha_2} \right],$$

where $\Delta \rho_{2,\text{Fr}} = \sqrt{z_2/k}$. \quad (46)

Here, $\Delta \rho_2$ is the separation of points in the observer’s plane (and, equivalently, in the near phase screen at distance $z_2$).

The frequency–time correlation function can be obtained by the replacement $\Delta \rho_2 \to \Delta \rho_{2,\text{Fr}} + \Delta \rho_{2,\text{dif}}$, where

$$\Delta \rho_{2,\text{dif}} = -2z_1(\Delta f/v_0)\theta_{\text{ref,0}}, \quad (47)$$

$$\Delta \rho_{2,\text{dif}} = \text{V}_{\text{obs}} \Delta t. \quad (48)$$

Here, $\text{V}_{\text{obs}}$ is the observer’s velocity. Fig. 1 shows that the frequency structure of the diffraction pattern drifts in time, with speed $|\text{d}f/\text{d}t| = 1.5 \text{ MHz/1000 s}$, so that diffraction spots are extended correspondingly in the dynamic spectrum. The component of velocity $\text{V}_{\text{obs}}$ parallel to the refraction angle $\theta_{\text{ref,0}}$ defines this drift. It produces a shift with the time lag of the minimum in the frequency of the structure function, as shown in Fig. 9. The component of velocity perpendicular to $\theta_{\text{ref,0}}$ does not contribute to the shift, but does produce an asymmetry of the structure function about the frequency lag of the minimum, $\text{V}_{\text{obs}}$. Specifically, the structure function becomes flatter for frequency lags smaller than $\text{V}_{\text{obs}}$ and steeper for frequency lags larger than $\text{V}_{\text{obs}}$. It is difficult to distinguish such an asymmetry at large time shifts because of the strong influence of noise. However, when $\text{V}_{\text{obs}}$ is parallel to the refraction angle $\theta_{\text{ref,0}}$, features in the dynamic spectra are strongly elongated, as we observe. This suggests that the perpendicular component of velocity is small compared with the parallel component and we conclude that the vectors $\text{V}_{\text{obs}}$ and $\theta_{\text{ref,0}}$ are approximately parallel. In this case, we can represent the frequency–time correlation function as

$$R(\Delta \rho_2) = D_{5,2}(\Delta \rho_{2,\text{Fr}}) \left[ 1 - \frac{1}{2} \left( \frac{1}{f_{2,0}} - \frac{f}{f_{2,0}} \right)^{\alpha_2} \right]. \quad (49)$$

From the shift of $\text{V}_{\text{obs}}$ for the largest time shift of $\Delta t = 640 P_1$ (Fig. 9), $f_{2,0} = 1.1 \text{ MHz}$, we find $f_{2,0} = f_{2,0}(\text{d}f/\text{d}t)^{-1} = 700 \text{ s}$.

The observer’s velocity relative to the Local Standard of Rest, perpendicular to the pulsar’s line of sight for the date of our observation, was $\text{V}_{\text{obs}} = 23.7 \text{ km s}^{-1}$. Using this value, we find for the Fresnel scale $\Delta \rho_{2,\text{Fr}} = 2^{1/\alpha_2} \text{V}_{\text{obs}} f_{2,0} = 2.5 \times 10^5 \text{ cm}$, using $\alpha_2 = 1.73$. The observer velocity is the vector sum of the velocity of orbital motion of the Earth on the date of observation and the velocity of the Sun relative to the Local Standard of Rest projected perpendicular to the pulsar’s line of sight. We do not know the velocity of the clouds of turbulent plasma responsible for scintillation, but we assume they have a velocity relative to the observer of $|\text{V}_{\text{scat}}| \approx 10 \text{ km s}^{-1}$ or less, although we do not know its direction and magnitude. The Fresnel scale we find corresponds to $\text{V}_{\text{scat}} = 0$. If we assume that the screen has a velocity of 10 km s$^{-1}$, the error in the evaluation of $\Delta \rho_{2,\text{Fr}}$ will be $0.75 \times 10^5 \text{ cm}$ (about 30 per cent). Accordingly, the distance to the near layer is $z_2 = 0.14 \pm 0.05 \text{ pc}$. Using equations (47) and (49) with $\Delta f = f_{2,0}$, we obtain $\theta_{\text{ref,0}} = 110 \pm 30 \text{ mas}$.

We found above that the relative amplitude of the second component is 0.15. Hence,

$$D_{5,2}(\Delta \rho_{2,\text{Fr}}) = (k \theta_{\text{scat,2}} \Delta \rho_{2,\text{Fr}})^{\alpha_2} = 0.15. \quad (50)$$

For $\alpha_2 = 1.73$, we find $\theta_{\text{scat,2}} = 0.4 \text{ mas}$. The error in $\theta_{\text{scat,2}}$ is about 30 per cent.

From the condition that the cosmic prism has no significant effect on the frequency correlation of diffractive scintillations, we can estimate an upper limit for the distance to the cosmic prism, $z_{\text{cosm}}$. The change in the refraction angle of the cosmic prism with a
change of frequency $f$ displaces the diffraction pattern from the strongly scattering far screen. We assume that the frequency scale of the pattern $\Delta_{\text{dif}}$ is less than the offset of the scattering from refraction. We then obtain the displacement

$$2(\Delta_{\text{dif}}/\nu_0)z_{\text{prism}}\theta_{\text{ref},0} < \Delta_{\text{dif}} = 4.6 \times 10^6 \text{ cm}. \quad (51)$$

Substituting our observed values of $\Delta_{\text{dif}} \approx 330 \text{ kHz}$ and $\theta_{\text{ref},0} = 110 \text{ mas}$ into this inequality, we find $z_{\text{prism}} \leq 1.4 \text{ pc}$, or $z_{\text{prism}} \leq 2 \text{ pc}$ when we include the error in our estimate of $\theta_{\text{ref}}$.

Thus, we find three components that contribute to scintillation of pulsar B1919+21: distant material at $z_1 \approx 440 \text{ pc}$, or material homogeneously distributed along the line of sight to the pulsar; a cosmic prism at a distance $z_2 \leq 2 \text{ pc}$ and a nearby screen at $z_2 = 0.14 \text{ pc}$. Most pulsars show some scintillating distributed along the line of sight and the distant material indicates that B1919+21 is no exception.

Cosmic prisms are seen for a number of nearby pulsars. Shishov et al. (2003) found the first evidence for such a component: from analysis of multifrequency observations they found a refraction angle in the direction to PSR B0329+54 of about 0.6 mas at frequency 1 GHz. They inferred that the size of the irregularities responsible for refraction is less than or about 3 $\times$ 10$^{-15}$ cm. Smirnova et al. (2006) found an indication that refractive effects dominate scattering for the direction to PSR J0437–4715. Smirnova et al. (2014) found a cosmic prism in the direction of PSR B0950+08 using space–ground interferometry. The refraction angle was measured as 1.1–4.4 mas at frequency 324 MHz. Here, we report the first localization of a cosmic prism, at a distance of about 1.4 pc in the direction to PSR B1919+21. The material associated with this prism is unknown. However, distances of only a few pc are inferred for the material that is responsible for the scintillation of intra-day variable extragalactic sources (Kedziora-Chudczer et al. 1997; Dennett-Thorpe & deBruyn 2002, 2003; Bignall et al. 2003, 2006; Jauncey et al. 2003). This material may lie at interfaces where nearby molecular clouds collide (Linsky, Rickett & Readfield 2008).

The distance that we find for the close screen, only 0.14 pc, is extremely close. It lies hundreds of times further away than the termination shock of the solar wind, but within the Oort cloud and hence within our Solar system. Our observation is the first detection of scattering by ionized gas in this region. Clearly, additional observations are needed to clarify the position and distribution of this material and its relation to other plasma components of the Solar system and the solar neighbourhood.

### 6 CONCLUSION

We have successfully conducted space–ground observations of PSR B1919+21 at frequency 324 MHz with a projected space–ground baseline of 60000 km. Analysis of frequency and time correlation functions and structure functions provides an estimate of the spatial distribution of interstellar plasma along the line of sight. We show that the observations indicate the existence of two components of scattering material in this direction. One is a screen located at a distance of about 440 pc from the observer, or distributed homogeneously along the line of sight. This shows strong diffractive scintillations and produces the largest effect. The second component is a much closer screen, at a distance of about 0.14 pc, which corresponds to weak scintillation. The Fresnel scale is equal to $2.5 \times 10^6 \text{ cm}$ at the near screen. Furthermore, a cosmic prism is located beyond the near screen, leading to a drift of the diffraction pattern across the dynamic spectrum at a speed $df/dt = 1.5 \text{ MHz}/1000 \text{ s}$. We have estimated the refraction angle of this prism as $\theta_{\text{ref},1} = 110 \text{ mas}$ and obtained an upper limit for the distance to the prism: $z_{\text{prism}} \leq 2 \text{ pc}$. Analysis of the spatial coherence function for the space–ground baseline (RA–GB) allowed us to estimate the scattering angle in the observer plane: $\theta_{\text{scat}} = 0.7 \text{ mas}$.

### ACKNOWLEDGEMENTS

The RadioAstron project is led by the Astro Space Center of the Lebedev Physical Institute of the Russian Academy of Sciences and the Lavochkin Scientific and Production Association under a contract with the Russian Federal Space Agency, in collaboration with partner organizations in Russia and other countries. We are very grateful to the staff at the Westerbork synthesis array and Green Bank observatory for their support. The study was supported by the program of the Russian Academy of sciences ‘Nonsteady and explosive processes in Astrophysics’. CRG acknowledges support from the US National Science Foundation (AST-1008865).

**Facilities:** RadioAstron Space Radio Telescope (Spektr-R), GB, WB.

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