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First Estimate of the Value of the Instrumental Polarization of the RadioAstron Space Radio Telescope Using the Results of an Early Scientific Program for Observing Active Galactic Nuclei

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Abstract—To interpret radio interferometric observations sensitive to linear polarization it is necessary to take into account the effect of instrumental or "parasitic" polarization. In the case of ground–space very-long-baseline interferometry, this procedure involves a session of polarization-sensitive mapping with the sufficient number of ground stations and sufficiently small projections of ground–space baselines. In the paper, the problem of estimating the value of the instrumental polarization of the *Spectr-R* space radio telescope of the RadioAstron project using the results of the early scientific program for observing active galactic nuclei is addressed. We use a statistical approach to estimate the value of the instrumental polarization associated with the analysis of already obtained to present time intermediate results of the review of brightness temperatures. Estimates obtained for the frequency bands *C* and *L* (95% probability intervals [0.0646, 0.1267] and [0.0945, 0.1736] for 6 and 18 cm, respectively) suggest that the instrumental polarization of the radio telescope does not exceed the values typical for the ground VLBI stations.

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1. INTRODUCTION

One of the main scientific goals of the RadioAstron project [1] is to perform ground–space radio interferometric observations of active galactic nuclei (AGN) with ultrahigh angular resolution, which are sensitive including to the linear polarization of the radio emission. Information about the polarization of the radio emission of magnetized plasma relativistic ejections and its frequency dependence is, in fact, a unique opportunity to study magnetic fields, elemental composition, and the energy spectrum of ejection matter particles. This is connected with the dependence of transfer coefficients of the polarized emission on these parameters [2, 3]. According to existing concepts, magnetic fields play an important role in the formation, collimation, and acceleration of jets in many space objects such as quasars, microquasars, and young stellar objects [4]. However, the problem of analyzing the results of polarization-sensitive observations using the method of very-long-baseline interferometry (VLBI) is complicated by the presence of instrumental effects, one of which is so-called polarization "leakage." Taking into account the weakness of the signal of the linear polarization of active galactic nuclei ejections (the degree of linear polarization in optically dense regions is typically only a few percent), ignoring this effect can lead to incorrect conclusions about the properties of the investigated objects. The unaccounted-for (or mistakenly accounted-for) effect of the instrumental polarization in magnitude and its location in the region of the maximum full intensity can coincide with the expected signal [5], which, in this case, makes interpretation extremely difficult.

It should be noted that the methods of estimates of the value of the instrumental polarization in the arsenal of ground VLBI are also applicable, as the experience of the VSOP project shows [6], in the case of ground–space VLBI. However, the accuracy of estimates obtained by these methods in connection with small experience of using the ground–space VLBI for polarization measurements have not been sufficiently studied. In particular, this applies to the case, in which in this frequency band the orbital radio telescope is able to detect only one orthogonal polarization component (as, for example, the *HALCA* Japanese satellite of the VSOP project [6] or *Spektr-R* in the frequency band of 6 cm). We performed a series of experiments with the observational data of the MOJAVE project [7] obtained using the VLBI network of the Very Long Baseline Array (VLBA) and taken by us from the standard archive, simulating the situation arising when calibrating the instrumental polarization of a space radio telescope (SRT). Within these experiments we have been considered the influence of two factors theoretically serving as a source of uncertainty when obtaining estimates of leakage, namely: the absence of intermediate-sized baselines with ground radio telescopes and recording only one of the orthogonal (in this case, circular) polarization components. The results of the performed experiments indicate that, in the case of the influence of both factors, the estimation accuracy of the instrumental polarization can be reduced by an order of magnitude.

In this paper, we solve the problem of obtaining a first estimate of the SRT instrumental polarization using the data of early scientific program (ESP) for observing AGNs, namely, the review of brightness temperatures.

2. LINEAR MODEL OF THE INSTRUMENTAL POLARIZATION

The signals induced in both polarizers of the radio telescope antenna, each of which is intended to receive one of the two orthogonal polarizations (in our case, the left and right circular polarizations) are actually the sum of two components: the response to the nominal and the response to orthogonal to it polarization:

$$V_R = G_R(E_R\exp(-i\varphi) + D_R E_L\exp(+i\varphi)),$$

$$V_L = G_L(E_L\exp(-i\varphi) + D_L E_R\exp(-i\varphi)),$$

where E_R and E_L are electric fields of the right and left orthogonal components of circular polarization of the radiation field, G_R and G_L are time-dependent complex antenna gain coefficients ($G_{iR}(t) = g_{iR}(t)\exp(i\psi_{iR}(t))$ for an antenna with the number i), D_R and D_L are complex coefficients (also designated as "D-member") that characterize the response to radiation of orthogonal polarization and are a quantitative measure of the effect of "leakage" and φ is the parallax angle of the polarizer, which characterizes its orientation relative to the observed source. It is important that the parallax angle changes continuously when accompanying the source by the antenna having altazimuth mounting, but remains constant in the case of an antenna with an equatorial mounting. This makes it possible, in many cases, to separate the signal of the parasitic polarization and the source signal.

It can be shown that the cross-correlations of responses of a pair of antennas are associated as follows with the Stokes parameters (visibility functions or amplitude of corresponding interferometric lobes) of emission of investigated source $\tilde{I}_{12}$, $\tilde{Q}_{12}$, $\tilde{U}_{12}$, $\tilde{V}_{12}$ and instrumental coefficients G_{1R}, G_{1L}, G_{2R}, G_{2R} and D_{1R}, D_{1L}, D_{2R}, D_{2R} [8]:

$$R_1R_2^* \equiv \langle V_{R1}V_{R2}^*\rangle$$

$$= G_{1R}G_{2R}^*\Big[(\tilde{I}_{12} + \tilde{V}_{12})\exp(i(-\varphi_1 + \varphi_2))$$

$$+ D_{1R}D_{2R}^*(\tilde{I}_{12} - \tilde{V}_{12})\exp(i(-\varphi_1 + \varphi_2))$$

$$+D_{1R}\tilde{P}_{21}^*\exp(i(-\varphi_1 + \varphi_2)) + D_{2R}^*\tilde{P}_{12}\exp(i(-\varphi_1 - \varphi_2))\Big],$$

$$L_1L_2^* \equiv \langle V_{L1}V_{L2}^*\rangle$$

$$= G_{1L}G_{2L}^*[(\tilde{I}_{12} - \tilde{V}_{12})\exp(i(+\varphi_1 - \varphi_2))$$

$$+ D_{1L}D_{2L}^*(\tilde{I}_{12} + \tilde{V}_{12})\exp(i(-\varphi_1 + \varphi_2))$$

$$+ D_{1L}\tilde{P}_{12}^*\exp(i(-\varphi_1 - \varphi_2)) + D_{2L}^*\tilde{P}_{12}^*\exp(i(+\varphi_1 + \varphi_2)).$$

$$R_1L_2^* \equiv \langle V_{R1}V_{L2}^*\rangle$$

$$= G_{1R}G_{2L}^*\Big[\tilde{P}_{12}\exp(i(-\varphi_1 - \varphi_2))$$

$$+ D_{1R}D_{2L}^*\tilde{P}_{12}^*\exp(i(+\varphi_1 + \varphi_2))$$

$$+ D_{1R}(\tilde{I}_{12} - \tilde{V}_{12})\exp(i(+\varphi_1 - \varphi_2))$$

$$+ D_{2R}^*(\tilde{I}_{12} + \tilde{V}_{12})\exp(i(-\varphi_1 + \varphi_2))\Big],$$

$$L_1R_2^* \equiv \langle V_{L1}V_{R2}^*\rangle$$

$$= G_{1L}G_{2R}^*[\tilde{P}_{21}\exp(i(+\varphi_1 + \varphi_2))$$

$$+ D_{1L}D_{2R}^*\tilde{P}_{12}\exp(i(-\varphi_1 - \varphi_2))$$

$$+ D_{1L}(\tilde{I}_{12} + \tilde{V}_{12})\exp(i(-\varphi_1 + \varphi_2))$$

$$+ D_{2R}^*(\tilde{I}_{12} - \tilde{V}_{12})\exp(i(+\varphi_1 - \varphi_2)).$$

Here, $\tilde{P}_{12} = \tilde{Q}_{12} + i\tilde{U}_{12} = \langle E_{1R}E_{2L}^*\rangle$ is the complex linear source polarization. The tilde over the Stokes parameter designates the value in the spatial frequency plane, but not in the image plane, and the asterisk designates the operation of complex conjugation. However, in practice, linearized ratios are often used (taking into account the small value of the degree of linear and especially circular source polarization, as well as D-members, about a few percent):

$$R_1R_2^* = G_{1R}G_{2R}^*\tilde{I}_{12}\exp(i(-\varphi_1 + \varphi_2)),$$

$$L_1L_2^* = G_{1L}G_{2L}^*\tilde{I}_{12}\exp(i(+\varphi_1 - \varphi_2)),$$

$$R_1L_2^* = G_{1R}G_{2L}^*[\tilde{P}_{12}\exp(i(-\varphi_1 - \varphi_2))$$

$$+D_{1R}\tilde{I}_{12}\exp(i(+\varphi_1 - \varphi_2)) + D_{2L}^*\tilde{I}_{12}\exp(i(-\varphi_1 + \varphi_2))].$$

$$L_1R_2^* = G_{1L}G_{2R}^*[\tilde{P}_{21}\exp(i(+\varphi_1 + \varphi_2))$$

$$+D_{1L}\tilde{I}_{12}\exp(i(-\varphi_1 + \varphi_2)) + D_{2L}^*\tilde{I}_{12}\exp(i(+\varphi_1 - \varphi_2))].$$

3. FORMULATION OF A PROBABILITY MODEL

Let us pass to the formulation of a probability model that connects the observational data and unknown instrumental coefficients. For this, it is necessary to construct a joint distribution of the observational data and unknown parameters, including both instrumental effects and the parameters of the observed source. Using the constructed model, we can estimate the probability density of the unknown parameters, using the Bayesian approach. We consider separately observational data and model parameters.

3.1. Used observational data. The main observational mode in the RadioAstron project within ESP for observing AGN is sufficiently short-time observational sessions with different a value and projection direction of a ground–space baseline. The obtained values for the visibility function after amplitude calibration or their upper limits are to be used for estimating the correlated flux and, therefore, the source brightness temperature [9]. The intermediate result of a review of brightness temperatures is the set of values of the signal/noise (S/N) ratios for the measured interferometric responses for various observational sessions. It follows from section 1 that in each lobe measurement of cross correlation (i.e., $R_1L_2^*$ or $L_1R_2^*$) for a baseline with SRT, there is a contribution from the instrumental polarizations for both baseline antennas. Taking into account the proposed constancy of coefficients of the SRT instrumental polarization, the intermediate results of the current review of brightness temperatures can be used for estimating these values.

It is possible to significantly reduce the number of unknown parameters in the model correlations connecting measurements in the review correlations with instrumental coefficients, forming the following ratios of cross to parallel correlations [8]:

$$\begin{aligned} &R_1L_2^*/R_1R_2^* \\ &= \frac{1}{r_2}e^{+i\Psi_2}(\tilde{M}_{12}\exp(-2i\varphi_2) + D_{1R}\exp(+2i\varphi_{12}) + D_{2L}^*), \\ &L_1R_2^*/R_1R_2^* = \frac{1}{r_1}\exp(-2i\Psi_1) \\ &\times\left(\tilde{M}_{21}^*\exp(+2i\varphi_2) + D_{1L}\exp(-2i\varphi_{12}) + D_{2R}^*\right), \\ &L_1R_2^*/R_1R_2^* = \frac{1}{r_2}\exp(-i\Psi_1)\Big(\tilde{M}_{21}^*\exp(-2i\varphi_1) \\ &+ D_{1L} + D_{2R}^*\exp(+2i\varphi_{12})\Big), \\ &L_1R_2^*/L_1L_2^* = r_2\exp(-i\Psi_2)\Big(\tilde{M}_{21}^*\exp(+2i\varphi_2) \\ &+ D_{1L}\exp(-2i\varphi_{12}) + D_{2R}^*\Big), \end{aligned} \quad (1)$$

where $r_i = g_{1R}/g_{iL}$ is the ratio of amplitudes of gain coefficients for the left and right channels of the antenna with the number i, $\Psi_i = \psi_{iR} - \psi_{iL}$ is the difference between the corresponding phases, φ_{12} is the difference of the parallax angles for both antennas, $\tilde{M}_{12}$ is complex degree of the linear polarization of the observed source. Here, m is the degree and χ is the position angle of the linear source polarization. We will assume that the sensitivities of both channels (left and right circular polarizations) of both radio telescopes formed the baseline are equal, which, strictly speaking, cannot be true. Then, the ratio of the amplitudes of interferometric lobes with cross to parallel correlations can be replaced by the signal/noise ratio for lobes of the corresponding correlations, i.e., to put $\left|R_1L_2^*\right|/\left|R_1R_2^*\right| = (S/N_{RL})/(S/N_{RR})$. Thus, using the ratio of the measured data, we eliminate the need for amplitude calibration. The model ratio connecting unknown parameters r, M, D_{GRT}, φ_i, and D_{RA} with the observed data $((S/N)_+/(S/N)_\|$ is as follows:

$$\begin{aligned} &(S/N)_+/(S/N)_\| \\ &= r\|M\exp(i\varphi_1) + D_{GRT}\exp((i\varphi_2) + D_{RA}\exp(i\varphi_3))\|, \end{aligned} \quad (2)$$

where $(S/N)_+$ is the signal/noise ratio for the cross correlation lobe (or upper limit for it), $(S/N)_\|$ is the same, but for the parallel correlation, r is the ratio of the amplitudes of the gain coefficients for the ground antenna, M is the degree of the linear polarization of the source, D_{GRT} is the amplitude of the coefficient of the instrumental polarization for the ground antenna, D_{RA} is the same for RadioAstron, φ_i are phases ($i = 1, 2, 3$). Table 1 shows the distribution and their parameters.

Using the intermediate results of ESP for observing AGN, namely, the correlated data of the review of brightness temperatures, we estimated the signal/noise ratio for lobes of cross correlations in the PIMA software package [10]. Further, we considered only scans with the detected signal (interferometric lobe) in one of the parallel correlation having the signal/noise ratio $S/N > 30$. For each session, consisting of several 10-min scans, the signal/noise ratios for the lobes of cross and parallel correlations were averaged over the scans (in the case of the detected signal, the criterion of which was the ratio of $S/N > 5.7$). In the presence in the session of only upper limits, the lowest upper limit was chosen to be used in the future. Table 2 presents a summary of the data thus obtained. We have not estimated the leakage in the K band in connection with the insufficient number of interferometric lobes with necessary S/N detected in this band.

3.2. Unknown parameters. In the Bayesian approach, all unknown parameters r, M, D_{GRT}, φ_i of the ratio (2) used by us, except SRT leakage amplitude, which is interesting for us, are usually included in the model as so-called nuisance parameters. These are the parameters required to construct the model, over which integrations are performed in the end. In our case, in particular, taking into account the inconstancy of these parameters on time scales much

Table 1

Physical value	Designation	Used distribution	Parameters
Ratio of the leakage amplitude in the right and left bands	r	Lognormal $\log N$(m, s)	$\mu = 0$, $\sigma = 0.25$
Amplitude of the complex degree of the source polarization	M	Generalized (or four-parametric) beta distribution $B(M_{\min}, M_{\max}, a, b)$	$M_{\min} = 0.0$, $M_{\max} = 0.20$, $a = 2$, $b = 3$
Amplitude of the leakage coefficient of the ground antenna	D_{GRT}	Generalized (or four-parametric) beta distribution $B(D_{\min}, D_{\max}, a, b)$	$D_{\min} = 0.01$, $D_{\max} = 0.2$, $a = 5$, $b = 5$
Phase members	φ_i, $i = 1, 2, 3$	Uniform $U(\varphi_{\min}, \varphi_{\max})$	$\varphi_{\min} = -\pi$, $\varphi_{\max} = \pi$

shorter than the ESP duration, this approach would lead to a significant increase in the number of model parameters. Thus, coefficients of the instrumental polarization of the ground antennas usually vary with changes in the configuration of polarizers/receivers, for example, in connection with the next stage of their service. In the presence of N estimates of the ratios of S/N lobes of the cross and parallel correlations (including upper limits for these ratios), the model will contain $5N + 1$ nuisance parameter and the interesting for us amplitude of the D_{RA} coefficient of the SRT instrumental polarization. Basically, because of the possibility to indicate a priori sufficiently close constraints on the values of nuisance parameters, the estimate of interesting for us amplitudes of the SRT leakage coefficients could perform in such manner. However, we used a different approach, in which the nuisance parameters were changed by the distributions of the corresponding values. The parameters of these distributions were fixed in accordance with the limitations discussed below.

3.3. The choice of parameters for the model distributions. Fixed parameters of the model distributions of r, M, D_{GRT}, φ_i were chosen on the basis of the limitations following from both the theoretical analysis and the previous experimental results. Thus, the amplitudes of coefficients of the instrumental polarization of the VLBI ground stations are usually of about 5% or less in the case of the VLBI network of VLBA, and up to 15–20% for some radio telescopes of the European VLBI Network, EVN. Taking into account this moment, it is possible to limit the amplitudes of the D_{GRT} leakage coefficients for ground stations by the value of 20%. The situation with the value of M degree of the linear source polarization is less certain. On the scales of the VLBI resolution, there are very few unpolarized sources. As the experience of the VSOP space interferometer project shows, the "VLBI core" observed from the Earth[1] is the result of blending the "real" optically thick "core" and more polarized close VLBI component resolved during VLBI observations with ground–space baselines [11]. Since for the majority of observations according to EST the implemented resolution corresponds to baselines in several times larger than the *HALCA* satellite orbit of the VSOP project [11], we can assume that the total detectable on the baselines with SRT flux corresponds to an optically thick "core." Taking into account the small values of the degree of the linear polarization of optically thick synchrotron source [2], it is possible to limit the distribution of the degree of the linear polarization by the value of 5%. Further, taking into account the randomness of the phase of the complex degree of the polarization of each observed source depending on the structure of the magnetic field for ejection from a particular source and a variety of other factors, as well as the constantly varying parallax angle of *Spektr-R* due to the SRT turns, we can assume that all phases φ_i in the model ratios are random values selected from the distribution uniform on the interval $[-\pi, \pi]$. Strictly speaking, the phase of the coefficient of the SRT instrumental polarization as the amplitude is expected to be constant, which, in fact, allows us to perform such statistical analysis. However, taking into account that in the RL/LL ratio (see (1)), which is applicable to the frequency band of 6 cm, the SRT leakage coefficient enters with phase factor φ_{12} varying with time, the phase distribution φ_3 of corresponding summand in (2) can also be considered homogeneous. In the case of observations in the band of 18 cm, we used only the model ratios for cross and parallel correlations, which contain the SRT leakage coefficient also with the "random" phase summand.

It should be noted that at the selected parameters of the model distributions of D_{GRT} and M, the result of the D_{RA} estimate corresponds also to the possibility, in which the D_{GRT} parameters actually are the M parameters, i.e., the so-called nonidentification of these parameters is shown. This is expressed in the presence of several peaks in the likelihood function. However,

Table 2

Form of data	Band C	Band L
Number of detected lobes	15	9
Number of upper limits	11	2

[1] The VLBI core is designated optically thick base of usually single-sided ejections observed in the radio images obtained by the VLBI method.

these parameters are not evaluated, and integration is carried out over them.

For given amplitude D_{RA} of the SRT leakage coefficient and parameters of the model distributions $\boldsymbol{\theta}$ (see Table 2), the predicted data (the values of the signal/noise ratios for lobes of cross to parallel correlations) are a sample from the distribution of $R(D_{RA}, \boldsymbol{\theta})$ obtained in accordance with ratio (2). Then, the likelihood function of D_{RA}, $\boldsymbol{\theta}$ parameters for obtaining the data $\mathbf{y}$ (i.e., the probability of the data $\mathbf{y}$ obtained in ESP to be selected from the distribution of $R(D_{RA}, \boldsymbol{\theta})$) is given by the expression:

$$L(\mathbf{y}|(D_{RA}, \boldsymbol{\theta})) = \prod_{i=1}^{N_{det}} p(R = R_i) \cdot \prod_{j=1}^{N_{lim}} F(R < R_j),$$

where $p(R)$ and $F(R)$ are the probability density and the distribution function (integrated probability density) estimated from the predicted distribution of $R(D_{RA}, \boldsymbol{\theta})$ and the observed data $\mathbf{y}$, N_{det} is the number of detections for lobes of cross correlations, N_{lim} is the number of upper limits per the amplitude for lobes of cross correlations. Fixing the parameters of the model distributions of nuisance parameters $\boldsymbol{\theta}$ based on a priori information, we, thus, exclude them from the model parameters. That is, the likelihood can be written as $L(\mathbf{y}|(D_{RA}, \boldsymbol{\theta}))$, where I is the used model (with all parameters).

Thus, for estimating the required amplitude we come to the following probability model:

$$P(D_{RA}, \mathbf{y}, I) = L(\mathbf{y}|D_{RA}, I) Pr(D_{RA}, I) Pr(I),$$

where $Pr(D_{RA}, I)$ is a priori probability distribution of the SRT leakage amplitude for the model I, but $L(\mathbf{y}|D_{RA}, I)$ is the likelihood presented above. $Pr(I)$ is an a priori probability for the I model.

4. ESTIMATE OF LEAKAGE AMPLITUDE

According to Bayes' theorem [13], the a posteriori probability density of the SRT leakage amplitude is given by the ratio

$$P(D_{RA}|\mathbf{y}, I) = \frac{P(D_{RA}, \mathbf{y}, I)}{P(\mathbf{y}, I)} = \frac{L(\mathbf{y}|D_{RA}, I) Pr(D_{RA}, I)}{P(\mathbf{y}, I)},$$

where $P(\mathbf{y}, I)$ is the normalization constant, the so-called evidence or the model likelihood. The evidence is the probability of the observed variables (i.e., the data) after marginalization (integration) of all model parameters and is the main instrument for the Bayesian comparison of models. For further analysis as non-informative a prior distribution $Pr(D_{RA}, I)$, we selected a distribution uniform over the interval [0, 1]. Wherever necessary, the estimate of the corresponding densities is performed by the nonparametric Parzen window method with Gaussian core and the width of the window determined in accordance with the Scott rule [14].

To construct a sample from a posteriori distribution $P(D_{RA}|\mathbf{y}, I)$ we used the Markov Chained Monte Carlo, MCMC, or rather, its affine invariant implementation [15, 16]. Assembly chains were initialized in the areas of the D_{RA} parameter selected on the basis of a preliminary study of the likelihood $L(\mathbf{y}|D_{RA}, I)$. However, preliminarily a "burn" was carried out, after reset of the results of which the simulation was performed, so that the exact values of the chain initialization should not influence the result. To reduce the effect of the correlation between neighboring chain parameters on the obtained estimates we used only every tenth value of the obtained chain.

It should be noted that in order to obtain estimates of the SRT leakage amplitudes we could also use an estimate of the maximum likelihood. However, taking into account that, in the future, we will plan to estimate the polarization properties of the observed sources using the leakage estimates obtained when mapping, it seems reasonable to use the mechanism of Bayesian analysis, which allows us to take into account a priori information on unknown parameters and study the hierarchical models. Besides which, the sample obtained by MCMC from the a posteriori probability distribution of the leakage amplitude allows us sufficiently simply to perform a predictive analysis of the used model for the adequacy of the description of the observable data (see below).

The a posteriori probability distribution of the leakage amplitudes for both frequency bands was also estimated by direct numerical integration. Indeed, in the one-dimensional case the application of the MCMC method for constructing the distribution density may seem excessive, but to check the model, in any case, it is necessary to obtain a sample of corresponding posteriori density. In addition, the used MCMC algorithm does not require adjusting any parameters, which essentially simplifies its use. In both cases, direct integration gave a similar result.

Figures 1a and 1b show histograms of a posteriori density distribution of the D_{RA} amplitude of leakage coefficients for the frequency bands C (6 cm) (the coefficient of leakage of the right polarization to the left D_L) and L (18 cm) (the coefficient of leakage of the left polarization to the right D_R) with the limits of 95% probability intervals (dashed lines). When constructing histograms of densities, a method [20] was used that maximizes the a posteriori probability of the number of cells of the histogram in the case of the piecewise constant density model implemented in [21]. Table 3 shows the corresponding average distributions, as well as their 95% probability intervals.

5. DISCUSSION

It should be noted that, in the solution to the problem, there are unaccounted-for sources of uncertainty. Thus, the probability model used is based on the linear model of leakage. However, we obtained a result in

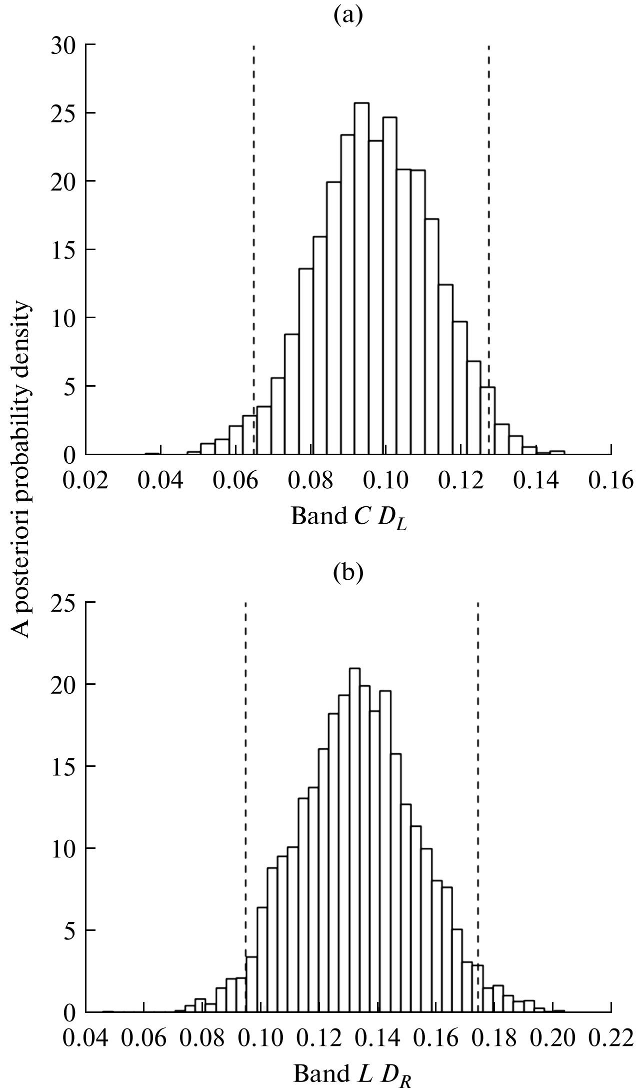

Fig. 1. Histogram of a posteriori distribution $P(D_{RA}|\mathbf{y})$ of the amplitude of the coefficient of the instrumental polarization D_L for the C and L frequency bands. To construct histograms 5000 points were used, which were obtained from $P(D_{RA}|\mathbf{y})$ by the Monte Carlo Markov chains.

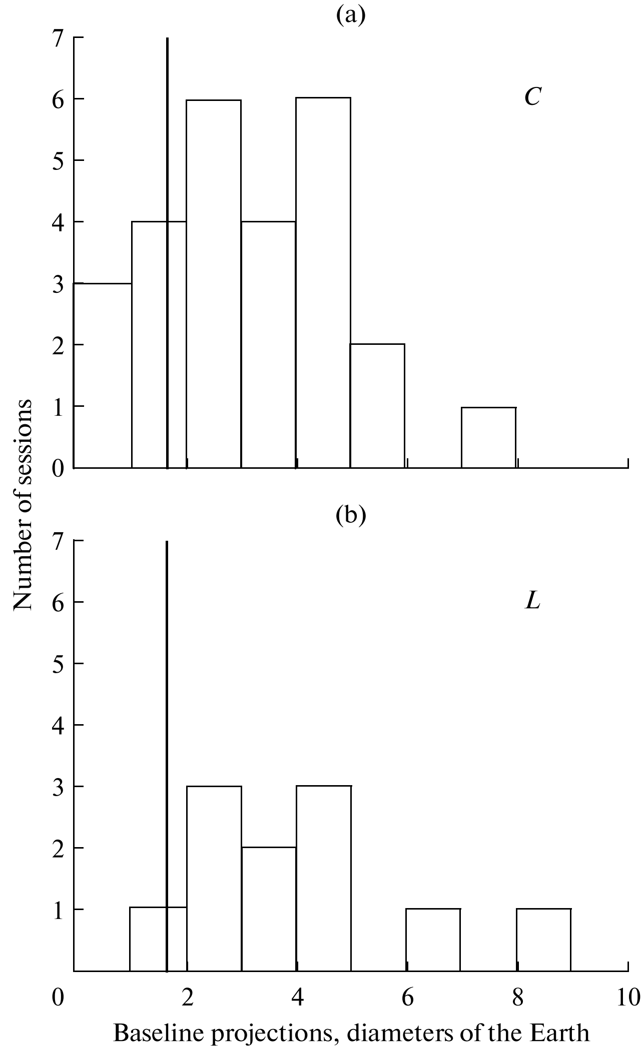

Fig. 2. Histogram of distribution of lengths of baseline projections used in observational sessions in the C and L bands. Bold vertical line corresponds to apogee of orbit for the *HALCA* space radio telescope of the VSOP project.

magnitude greater than the value after which when estimating leakage by the standard for VLBI methods terms of the second order of smallness are added in the linear model. But, since we are interested in how we cannot better estimate the effect of leakage in order to interpret a particular polarization experiment, but rather estimate the value of the effect, using the linear model seems justified. Moreover, when using a nonlinear model of leakage, a sufficiently simple (in view of the number of parameters) formulation of the probabilistic model is found to be impossible. We plan to return to the formulation of a more complete model after finishing the review of the AGN brightness temperatures and determining the value of leakage by the standard for VLBI methods in order to, for example, perform a statistical study of the polarization properties of observed objects on the resolution scales of corresponding ground–space baselines with SRT.

Table 3. Parameters of obtained samples from a posteriori distributions of the amplitudes of coefficients of the instrumental polarization

Parameter	Average value	95% probability interval
Band C	0.0968	[0.0646, 0.1267]
Band L	0.1328	[0.0945, 0.1736]

Further, in the implemented approach, the fixed parameters of the distributions $\boldsymbol{\theta}$ used to construct the probability model, in fact, are the structural model parameters and must be separately determined, before estimating the required leakage amplitude D_{RA}. We

Table 4. Values of the evidence for considered models. The *C* band

Boundaries of distributions $B(2, 3)$	$\ln Z \pm \ln Z$	$\ln Z - \ln Z_{\min}$	$B_{i\text{-min}}$
[0.00, 0.05]	16.3678 ± 0.1794	3.9896	54.0332
[0.00, 0.10]	15.9771 ± 0.1694	3.5989	36.5580
[0.00, 0.20]	15.5265 ± 0.1565	3.1483	23.2964
[0.05, 0.20]	15.3821 ± 0.2159	3.0040	20.1660
[0.10, 0.20]	12.3782 ± 0.2230	0	—
see Fig. 3	15.5640 ± 0.1569	3.1858	24.1870

fixed their values on the basis of the limitations following from both theoretical considerations (in the case of the maximum value of the degree of polarization) and previous experimental results (the results of the VSOP project and available information on the leakage amplitudes of ground stations). However, if the distribution of the amplitudes of leakage coefficients D_{GRT} of ground stations can be made sure, estimation the distribution of the degree of polarization M of observed sources cannot be considered final. The fact is that it is based on the data from ground-based VLBI observations and the data of ground-space interferometer of the VSOP project. Data used in the paper were obtained with an angular resolution several times better than that achieved in the VSOP project (see Fig. 2). In addition, we use the idea of a "VLBI core" as the basis of an optically thick ejection base that is the model assumption, which, although is supported by the ground-based data, cannot be consistent with observational data for ultra-high resolution of the ground–space VLBI with the RadioAstron SRT.

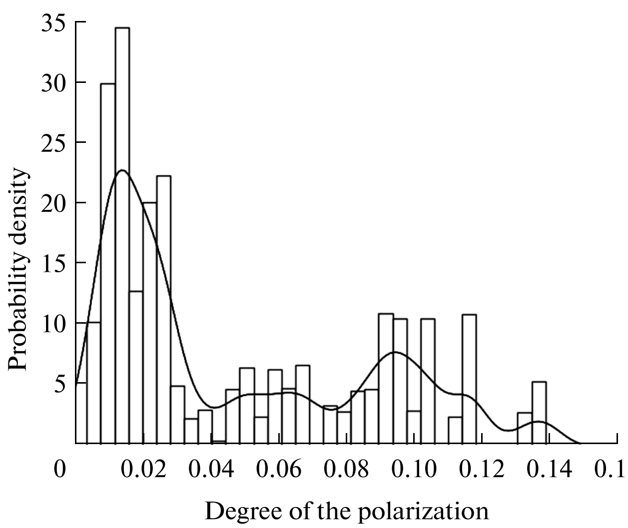

Fig. 3. Histogram of distribution of observed degree of linear polarization in sources investigated in this work according to the MOJAVE review at 15 GHz. It was used 1000 subsamples with size matched with the size of the sample used in this work (only detected signal) in the *C* band. Each subsample included the polarization degree of the MOJAVE archive for the last 3 years for each source selected randomly in proportion to the number of sessions of particular source used in an analysis. Solid line shows probability density obtained by corer method of estimation with Gaussian core (see text).

We could continue the analysis in this direction, for example, considering the parameters of the distributions of the degree of the polarization of the observed sources as "latent" parameters and giving for them any a priori distributions, to estimate together with the SRT leakage amplitude. Or choosing the parameters of model distributions (or even the type of distributions) maximizing the evidence of the model to estimate the a posteriori distributions $P(D_{RA}|\mathbf{y}, I)$, using the model already obtained on the basis of the selected distributions. This analysis was performed. We used a generalized beta distribution with different upper and lower limits for the simulation of possible situations described as a standard optically thick "core," bright strongly polarized component with the degree of polarization of up to 20%, as well as the intermediate cases. In addition, we use an empirical distribution obtained on the basis of the review of the MOJAVE data [7] for the degree of the polarization of the sources of observed sample for the last 3 years (see Fig. 3). Although the MOJAVE data were obtained for the VLBI network of VLBA at a frequency of 15 GHz, using the empirical distribution can be considered reasonable, taking into account the fact that the characteristic size of cells of heterogeneity of the Faraday screen estimated according to the ground VLBI is not much less than synthesized at this resolution [17]. Therefore, with the RadioAstron resolution the Faraday depolarization on the external screen should be small.[2] An estimate of the evidence of each of the models was the byproduct of the MCMC sampling by the method of thermodynamic integration using the algorithm of parallel hardening.

Tables 4 and 5 show the obtained values of the evidence Z for the investigated set of models for the C and L band, respectively. The columns contain: boundaries of the beta distribution used in each of the models $B(2, 3)$ being the distribution of the degree of the polarization of the observed sources; the natural

[2] However, it should be remembered for one of the results discussed in section 3.2 of the VSOP project [10], according to which the VLBI core observed from Earth when mapping with the ground–space interferometer reveals close to the "true" VLBI core a bright strongly polarized component.

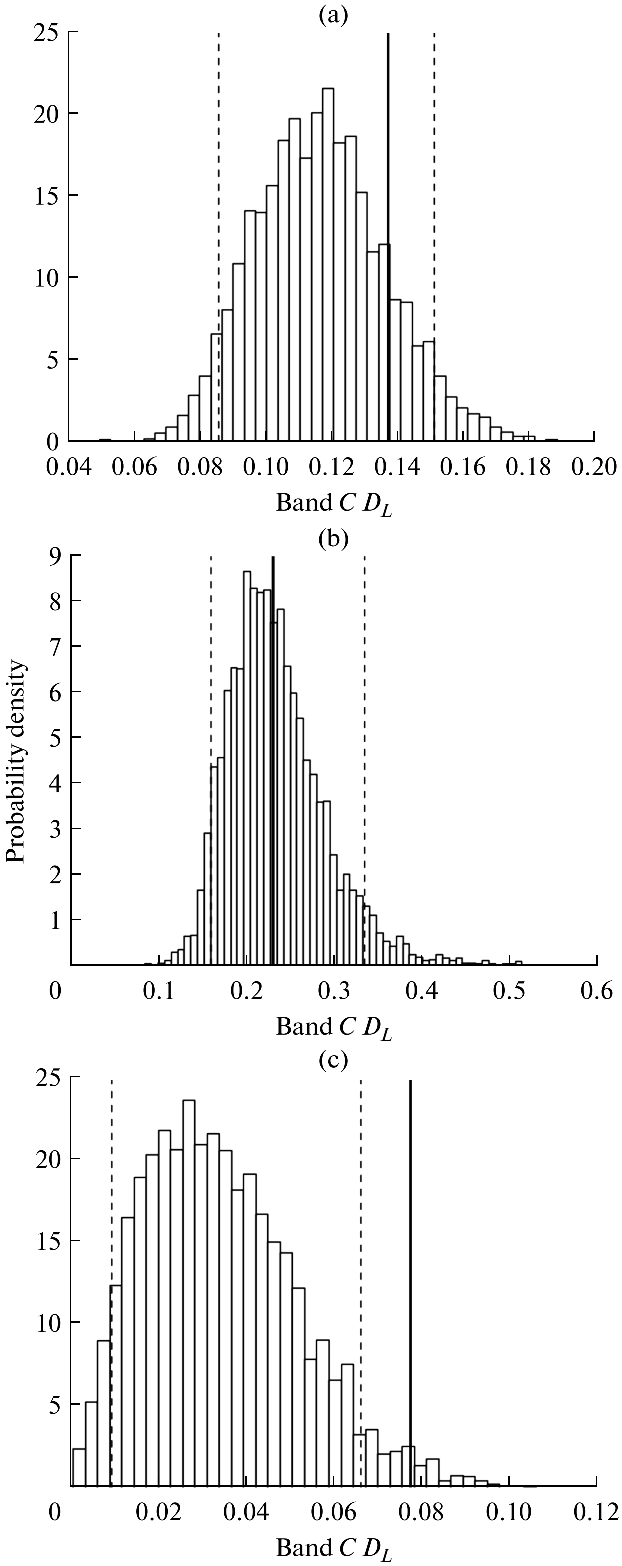

Fig. 4. Histogram of distributions of average (a), maximum (b), and minimum (c) values for 5000 sets of hypothetical "replication" data obtained on the basis used for analysis probability model and a sample from a posteriori distribution of the amplitude of leakage coefficient for the C frequency band. Dotted lines indicate the 5 and 95% levels. Solid line shows observational data.

Table 5. Values of the evidence for considered models. The L band

Boundaries of distributions $B(2, 3)$	$\ln Z \pm \ln Z$	$\ln Z - \ln Z_{\min}$	$B_{i\text{-min}}$
[0.00, 0.05]	6.9359 ± 0.1578	0	—
[0.00, 0.10]	7.1373 ± 0.1483	0.2014	1.2232
[0.00, 0.20]	7.3599 ± 0.1126	0.4240	1.5281
[0.05, 0.20]	7.5868 ± 0.1169	0.6509	1.9172
[0.10, 0.20]	7.0668 ± 0.1278	0.1309	1.1399
see Fig. 3	7.1217 ± 0.1456	0.1858	1.2041

logarithm $\ln Z$ of the evidence of the models and its error $d\ln Z$; removing each of the models from the model with a minimum evidence in the logarithmic scale; the Bayesian factor $B_{i\text{-min}}$ (the evidence ratio) of the models with respect to the model with the smallest evidence.

As can be seen, the too-small volume of the sample of the observational data used cannot confidently select the best model (models) from a set of models on the basis of ratios of the evidence of the models (the so-called "Bayesian factors") [18]. An exception, perhaps, is the evidence against the model described as "highly polarized component." The same results were obtained by direct numerical integration, as well as calculating the evidence of the Laplace approximation (fitting the likelihood function peak by the Gaussian function). Again, we are planning to return to the statistical study of the polarization of nuclei for ejections after the review of brightness temperatures and determination of the value of SRT leakage by methods already standard for VLBI.

To check the adequacy of the description of the observed data of the used probability model (including the linear approximation of leakage, the selected parameters of model distributions, a prior distribution of the leakage amplitude) we performed an a posteriori predicative check of the model as follows [19]. Using a sample from the a posteriori distribution of the leakage amplitude $D_{RA,\,i}$ obtained during MCMC and obtained in accordance with the generating model of the distribution of corresponding relations for the correlations $P(D_{RA}|\mathbf{y}, I)$ a set of the hypothetical replications for the data $\mathbf{y}_i^{repl}$ was composed. Further, the distribution of these statistics for the data replications was composed as data average, maximum, and minimum values. Then, it was checked how the "implemented" replications (i.e., the observational data) correspond to the constructed distributions of statistics. Corresponding distributions and observational data, as well as 5 and 95% boundaries, are shown in Figs. 4 and 5 for the average, minimum, and maximum values for the C and L bands, respectively. As can be seen from the figures, the "implemented" data are consistent with the distribution of the statistics for the replication data, except for the minimum value in the C band. Thus, in this band, the probability model used has difficulties

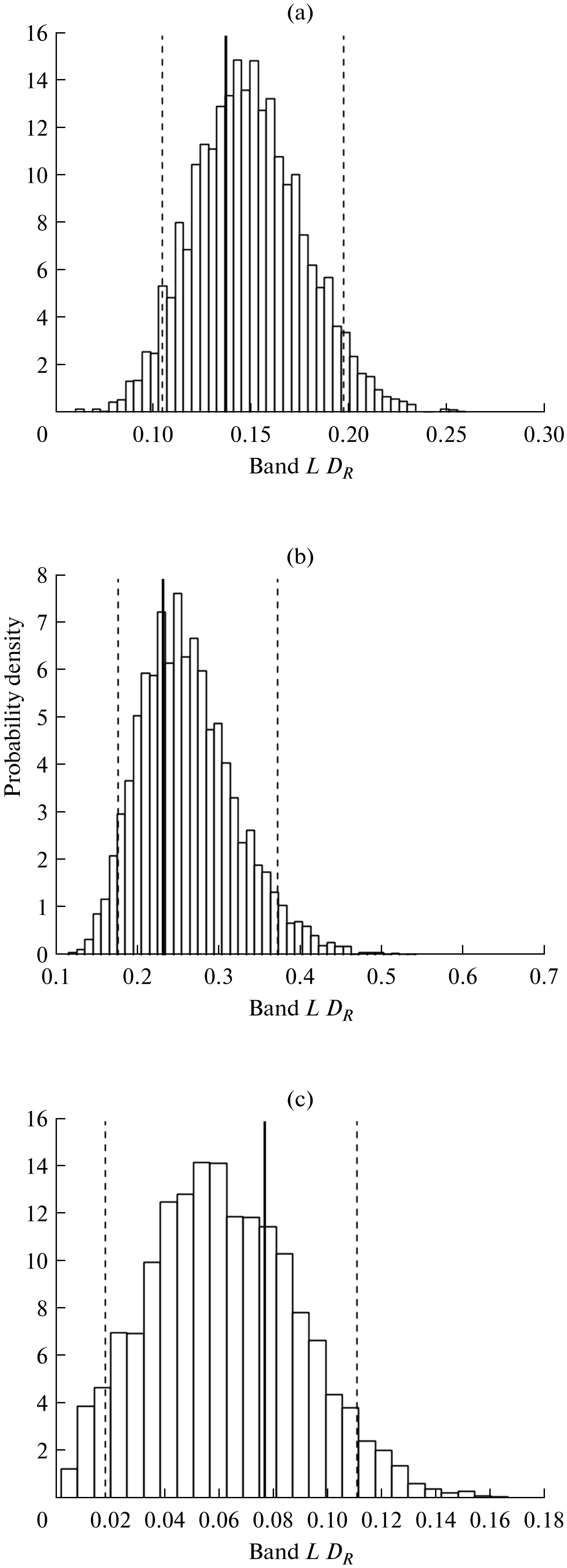

Fig. 5. Histogram of distributions of the average (*a*), maximum (*b*), and minimum (*c*) values for 5000 sets of the hypothetical "replication" data obtained on the basis used for the analysis probability model and sample from a posteriori distribution of amplitude of leakage coefficient for the L frequency band. Dotted lines indicate the 5 and 95% levels. Solid line shows observational data.

with an adequate description of the minimum values of the observational data. This can be connected with both the simplified linear model of leakage used by us and with used model distributions.

CONCLUSIONS

We carried out a statistical analysis of the ESP results for observing AGN, by which was shown that the effect of "leakage" of the polarization of the RadioAstron SRT does not exceed the values typical for some ground VLBI stations (95% probability interval [0.0646, 0.1267] and [0.0945, 0.1736] for 6 and 18 cm, respectively). It allows us to hope for successful implementation as already carried out in the framework of ESP, and planned within Key Scientific Programs experiments on polarization-sensitive mapping. Clarifying the corresponding estimates of the leakage coefficients by the standard for VLBI methods, when performing polarization sensitive mapping, allows us to use the data of the review of AGN brightness temperatures for statistical studies of the degree of the polarization of nuclei of compact radio sources at the ultra-high resolution in the different frequency bands.

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