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Trajectory Correction of the *Spektr-R* Spacecraft Motion

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Abstract—The results of refining the parameters of the *Spektr-R* spacecraft (RadioAstron project) motion after it was launched into the orbit of the Earth's artificial satellite in July 2011 showed that, at the beginning of 2013, the condition of staying in the Earth's shadow was violated. The duration of shading of the spacecraft exceeds the acceptable value (about 2 h). At the end of 2013 to the beginning of 2014, the ballistic lifetime of the spacecraft completed. Therefore, the question arose of how to correct the trajectory of the motion of the *Spektr-R* satellite using its onboard propulsion system. In this paper, the ballistic parameters that define the operation of onboard propulsion system when implementing the correction, and the ballistic characteristics of the orbital spacecraft motion before and after correction are presented.

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1. INTRODUCTION

In the standard case, the trajectory of the *Spektr-R* spacecraft is corrected in order to change the characteristics of its future flight into operational orbit of the Earth's artificial satellite, i.e., eliminating unwanted spacecraft setting in the shadow of the Earth or Moon (the light source is the Sun) and increasing the ballistic lifetime of the spacecraft.

By definition, the ballistic lifetime of the spacecraft at current time instant t is provided when it flies above the Earth's surface at no less than a given altitude $h_{\pi 1}$. Each time the spacecraft falls in the shadow of the Earth or Moon is characterized by the duration of its stay in full or partial shadow (half-shadow). The standard correction of the spacecraft motion into the operational orbit of the Earth's artificial satellite is executed by carrying out special correction sessions, in which the necessary spatial orientation of the thrust vector of the onboard propulsion system (PS) and the switching of the PS on and off are provided at given time instants. The cyclogram of the standard correction (correction scheme) of the operational orbit of the *Spektr-R* spacecraft is chosen in view of the technical spacecraft features, as well as the accuracy of our knowledge of the parameters of its motion and the technology of implementing the correction sessions. In this case, the total cost of the working body when PS operating should be close to the minimum.

2. CHARACTERISTICS OF PS AND THE SPACECRAFT CORRECTION SESSION

The calculation of the ballistic parameters necessary to choose the scheme for spacecraft trajectory correction, to implement and to analyze the correction execution is performed under the following assumptions on the technical spacecraft characteristics, the structure and the logic of corresponding session.

(1) At every time instant, the total thrust vector of operating PS (PS thrust) belongs to a line passing through the center of mass (CM) of the spacecraft.

(2) During the session, the spacecraft's motion in is correction by the continuous operation of the PS during the time interval, i.e., from the time instant $t_{\rm thn}$, designated as the time instant when the PS is switched on, to the time instant $t_{\rm the}$, designated as the time instant when PS is switched off.

(3) During the time interval $[t_{\rm thn}, t_{\rm the}]$ of PS operation, the thrust retains its direction in the inertial space. The unit vector of the thrust $\mathbf{e}_{\rm th}$ is considered as the vector collinear to a given or specially calculated vector $\mathbf{e}$ in the coordinate system (CS) of J2000 [1]. Hereafter, for convenience, it is assumed that, in the case, when vectors $\mathbf{e}_{\rm th}$ and $\mathbf{e}$ coincide in the direction, the value $V_{\rm ch}$ of the increment of characteristic velocity at the cost of the PS operation has nonnegative value. Otherwise, the value $V_{\rm ch}$ is taken with a minus sign. Thus, $V_{\rm ch} \geq 0$, if $\mathbf{e}_{\rm th} = +\mathbf{e}$, and $V_{\rm ch} < 0$, if $\mathbf{e}_{\rm th} = -\mathbf{e}$.

(4) The values of the thrust P and the specific PS pulse $I_{\rm sp}$ are constant for the entire time interval of its

continuous operation. Outside this interval, the PS thrust is absent, i.e., $P = 0$.

(5) In the correction session, the PS thrust is switched off (the increment of characteristic velocity of the spacecraft at the expense of PS is finished) after the Δt_{th} duration of its continuous operation, $\Delta t_{\text{th}} = t_{\text{the}} - t_{\text{thn}}$, is achieved.

(6) To implement the session for correcting the target spacecraft, it is sufficient to determine the necessary values t_{thn} and Δt_{th}, as well as vector $\mathbf{e}_{\text{th}}$ of the direction of the PS thrust in the J2000 CS, designated as the parameters of the spacecraft correction session.

3. DEFINITIONS AND ASSUMPTIONS IN THE PROBLEMS OF THE SPACECRAFT CORRECTION

The ballistic problems required for choosing the parameters of forthcoming correction of the spacecraft operational orbit or an analysis of the results of the performed spacecraft correction are considered under the following definitions and assumptions relative to the parameters that characterize the motion, as well as the light and shade situation for the spacecraft.

(1) When solving to the problem of ballistic support (BS) of the spacecraft flight control, the mathematical simulation of the motion the spacecraft CM is performed taking into account the attractive forces of the Sun, Moon, planets of the solar system considered as material points. Moreover, it is necessary to take into account forces caused by the noncentrality of the Earth's gravitational field [2], the aerodynamic resistance of the spacecraft motion in the Earth's atmosphere (the dynamic model of the atmosphere is used [3]), and light pressure on the spacecraft.

The acceleration $\mathbf{w}_{\text{a}}$ of the spacecraft caused by the atmospheric influence is calculated by the formula $\mathbf{w}_{\text{a}} = s\rho|\mathbf{V}_{\text{a}}|\mathbf{V}_{\text{a}}$, where ρ is the atmospheric density in the vicinity of the spacecraft, $\mathbf{V}_{\text{a}}$ is the spacecraft velocity relative to the atmospheric flow, and s is the so-called ballistic coefficient. The value of this coefficient depends on the dimensionless aerodynamic coefficient c_x, the midsection area S relative to the atmospheric flow and mass m of the spacecraft, $s = (c_x/2)(S/m)$. Taking into account that the spacecraft flight in operational orbit passes outside the dense layers of the Earth's atmosphere and that it rarely approaches these layers, s is taken as constant in ballistic calculations. The air density is calculated in full accordance with the dynamic model of the Earth's atmosphere. In this case, the input parameters of the model (the current level of the intensity of solar radiation, etc.) are overestimated with respect to the average, air density.

Generally speaking, the light pressure force is characterized by a dimensionless variable, i.e., value S_{d} of the ratio of the absolute value of indicated force to the attractive force of the spacecraft by the Sun. However, in the ballistic calculations of the spacecraft flight trajectories, for each specific trajectory, it is taken as constant and refined by the trajectory measurements and the telemetry (TM) information. It is considered to be a matching parameter that generally allows one to take into account forces that are small in the magnitude not simulated that act on the spacecraft when predicting the motion of the spacecraft CM.

(2) The trajectory of passive (without PS switching) spacecraft flight at each current time instant t is characterized by six-dimensional vector (x, y, z, V_x, V_y, V_z) of kinematic parameters of motion. The first three components of this vector are the coordinates of the position vector $\mathbf{r}(t) = (x(t), y(t), z(t))$ and the last three components are the coordinates of the vector $\mathbf{V}(t) = (V_x(t), V_y(t), V_z(t))$ of the spacecraft velocity in the J2000 CS. The collection of nine values $\{t, \mathbf{r}(t), \mathbf{V}(t), s, S_d\}$ are designated as the initial conditions (IC) of spacecraft motion at time instant t. It is used the decreed Moscow time (DMT), which is 3 h earlier than the corresponding Coordinated Universal Time **UTC**. It is assumed that the current flight spacecraft trajectory before PS switching is given by IC at the time instant t_0 before time instant t_{thn} of PS switching on as follows: $(t_0, x_0, y_0, z_0, V_{x0}, V_{y0}, V_{z0}, s, S_d)$.

(3) The value of the spacecraft mass m at the time instant t_{thn} of PS switching is known as follows: $m(t_{\text{thn}}) = m_0$.

(4) The time of continuous PS operation can be determined by three ways, i.e., explicitly based on the values t_{thn} and t_{the}; based on values t_{thn} and V_{ch} of the increment of the characteristic velocity as a result of the correction execution; and by the average time instant t^* of time segment of the PS operation, $t^* - t_{\text{thn}} = t_{\text{the}} - t^*$, and the value of increment of characteristic velocity V_{ch} as a result of the correction session execution. In the third case, the time instant t^* is usually found implicitly. It is determined as the first time instant (after a given time t_{g}) at which a certain condition on the kinematic parameters of the spacecraft motion is fulfilled. That time instant can be, e.g., the time instant of reaching the minimum (or maximum) distance between the satellite and the Earth's CM, assuming its passive flight. In this case, the trajectory is corrected at the pericenter (or apocenter) of the spacecraft orbit.

(5) The dependence between the duration of the PS operation and the corresponding increment of characteristic velocity is set by the Tsiolkovskii formula

$$\Delta t_{\text{th}}(V_{\text{ch}}) = (I_{\text{sp}}/P)g_0 m_0(1 - \exp[-|V_{\text{ch}}|/I_{\text{sp}}/g_0]), \quad (1)$$

where the thrust P and specific pulse I_{sp} are given by the PS parameters. Acceleration due to the force of gravity is taken equal to be $g_0 = 9.80665$ m/s^2. The parameters in formula (1) have the following dimensionalities: [s] for Δt_{th}, [s] for I_{sp}, [N] for P, and [kg] for m_0.

(6) The light and shade situation on the spacecraft at any fixed time instant is characterized by the dimensionless coefficient K_T of the Sun shading by the Earth (Moon), if looking from the spacecraft CM toward the Sun. The shading coefficient K_T is determined as follows based on the ratio of the area S_T of the shaded (hidden by the Earth or Moon) part of the Sun to the area S_S of the entire Sun visible from the spacecraft, assuming the absence of the Sun shading: $K_T = S_T/S_S$. It is assumed that the shapes of the Sun, Earth, and Moon are spheres with given radii. The geometric center of each sphere coincides with CM of the corresponding luminary. Obviously, the shading coefficient K_T can take the values that remain within the limits of the segment [0, 1] of the number axis. At $K_T = 0$, the spacecraft is in the light (there is no shading the Sun, Earth, or Moon). At $K_T = 1$, the spacecraft is in the shadow (there is a total solar eclipse, if to look from the spacecraft). At a numeric value of the coefficient of shading, which belongs to the set of interior points of the above segment, it can be assumed that the spacecraft is in the half-shadow (the partial solar eclipse occurs if one is looking from the spacecraft).

(7) By definition, the entire time segment of shading $[t_{shn}, t_{she}]$ on which the coefficient K_T is more than zero at each time instant t is characterized as follows by the coefficient $K_{T\max}$ of the degree of shading:

$$K_{T\max} = \max_{t \in [t_{shn}, t_{she}]} K_T(t).$$

(8) The time segment $[t_{ashn}, t_{ashe}]$ in which the equality $K_T = 1$ is designated as the time segment of the total solar eclipse at each time instant t. In this case, the segment (if it exists) is a unique subset of the corresponding segment of the time of shading $[t_{shn}, t_{she}]$.

(9) The prediction of the light and shade situation on the spacecraft after calculating the correction of its motion trajectory is reduced to the determination of a set of time segments of shading and the corresponding time segments of the total solar eclipse if they exist. The desired set of segments is caused by the parameters of the spacecraft trajectory correction and given time interval Δt_{sh} of the prediction of the light and shade situation on the spacecraft. The initial time instant Δt_{shn} for each desired segment of the time of shading should satisfy the inequality $t_{the} \le t_{shn} \le t_{off} + \Delta t_{sh}$.

(10) The lifetime of the spacecraft operational orbit is determined as the last time instant t_{le} that belongs to a given set of $M\{t_l\}$ of sequential time instants before which the following condition is fulfilled: the altitude $h_\pi(t_l)$ of the current spacecraft orbit exceeds given below the acceptable value $h_{\pi l}$ of the altitude of the spacecraft flight, i.e.,

$$h_\pi(t_l) \ge h_{\pi l}. \tag{2}$$

The indicated set represents a collection of individual time instants (q_l pieces) that belong to the given segment $[t_{l\min}, t_{l\max}]$ of the number axis, which is designated as the time segment of verifying the spacecraft lifetime as follows:

$$M\{t_l\} = \{t_{l\min}, t_{l\min} + h_{tl}, t_{l\min} + 2h_{tl}, \dots, t_{l\min} + q_l h_{tl}\}. \tag{3}$$

Here, $h_{tl} > 0$ is a given step of verifying the spacecraft lifetime (fulfilling condition (2)) and forming set (3), while q_l is determined by the time instant $t_{l\max}$, namely, the condition $t_{l\min} + q_l h_{tl} \le t_{l\max} < t_{l\min} + (q_l + 1)h_{tl}$. For definiteness, it is assumed that, if condition (2) is not fulfilled at the first point of set (3), then $t_{le} = t_{l\min}$.

3. SPACECRAFT ORIENTATION FOR IMPLEMENTING THE CORRECTION SESSION

This chapter is devoted to the ballistic analysis of the possibility of constructing the acceptable orientation for the spacecraft as a rigid body in order to implement the forthcoming correction session. In this case, it is assumed that the unit vector $\mathbf{e}_{th}$ of the PS thrust is a vector collinear to the vector $\mathbf{V}^*$ of the spacecraft velocity at the time instant t^* in the passive spacecraft flight $\mathbf{V}^* = \mathbf{V}(t^*)$.

The corresponding ballistic problem is presented in the chapter, as a result of which, when it is possible to construct the indicated spacecraft orientation, we obtain a solution containing values of its basic (orientation) parameters. We introduce the following right rectangular coordinate systems bound with the spacecraft: BCS $Ox_by_bz_b$, the center of which, point O, coincides with the spacecraft CM, and the Ox_b axis is directed along the vector of the PS thrust and BCS0, which, at time instant t^*, coincides with BCS provided that its Ox_b axis is directed along the vector $\mathbf{e}_{th}$, the Ox_bz_b coordinate plane contains the unit vector $\mathbf{e}_S$ of the direction from the spacecraft CM (the point O) to the Sun CM, and the positive direction of the Oz_b axis is an acute angle with the vector $\mathbf{e}_S$.

Before the session of correction spacecraft motion, the possibility of constructing the acceptable BCS orientation in the J2000 CS is verified, which is conditioned by the implementation of the restriction presented below.

Restriction. The angle γ between the direction of the Ox_b axis of BCS and the direction from the spacecraft to the Sun CM belongs to the given range:

$$\gamma_{\min} \le \gamma \le \gamma_{\max}, \tag{4}$$

where the boundaries of the range can be specified during flight and construction spacecraft tests and deviate from the values $\gamma_{\min} = 90°$ and $\gamma_{\max} = 165°$ within a few degrees, respectively.

In connection with the foregoing, the ballistic analysis of the possibility of constructing the spacecraft orientation in order to implement a correction session for its motion can be reduced to solving the following problem, which we will call the orientation analysis problem.

Orientation Analysis Problem

$t_0, x_0, y_0, z_0, V_{x0}, V_{y0}, V_{z0}, s, S_d$ are initial conditions of the spacecraft motion in the J2000 CS; t^* is middle of the interval of the PS operation of the correction session; Iv is the indicator of the PS thrust direction when the spacecraft is corrected (in the case of $Iv \geq 0$, the thrust is directed along the velocity $\mathbf{V}(t^*)$ and, in the case of $Iv < 0$, the thrust is directed against the indicated velocity); $\gamma_{\min}$, $\gamma_{\max}$ are the segment boundaries of acceptable angles γ between the PS thrust when making corrections and the direction from the spacecraft toward the Sun CM (see (4)).

Set: $\{\mathbf{r}(t^*), \mathbf{V}(t^*)\}$ are the kinematic parameters of spacecraft motion in the J2000 CS at the time instant when the spacecraft achieves (in passive flight) the point at which it is supposed to implement the session of the target correction of spacecraft motion;

the sequence of the parameters (r_S is the distance from the spacecraft to the Sun CM, $\mathbf{e}_S$ is the unit vector of the direction from the spacecraft to the Sun CM, r_E is the distance from the spacecraft to the Earth's CM, $\mathbf{e}_E$ is the unit vector of the direction from the spacecraft to the Earth's CM) that characterize the position of the Sun and the Earth relative to the spacecraft at the time instant t^*;

Io is an indicator of the possibility of constructing the orientation of the spacecraft as a rigid body to implement the session of correcting its motion; at $Io \geq 0$, it is possible to construct the necessary spacecraft orientation (inequalities (4) are fulfilled); at $Io < 0$, it is considered to be impossible to construct this orientation (inequalities (4) are not fulfilled) and the procedure for solving to the problem is completed; γ is the angle between the Ox_b axis of BCS directed along the PS thrust and the direction from the spacecraft to the Sun CM at the time instant t^*;

a sequence of unit vectors in the J2000 CS that corresponds to the directions of the BCS axis in its reference position of BCS0 (see above), i.e., $\mathbf{e}_{Ox}$ along the direction of the Ox_b axes, $\mathbf{e}_{Oy}$ along the direction of the Oy_b, and $\mathbf{e}_{Oz}$ along the direction of the Oz_b axis.

4. CHOOSING THE PARAMETERS FOR THE CORRECTION SESSION FOR THE TRAJECTORY OF THE SPACECRAFT

The characteristics of the correction session depend significantly on the time instant t^* (see above). At given values of mass m_0 before PS switching and the value V_{ch} of increment of characteristic velocity, it uniquely determines (using formula (1)) the time instants of switching the PS on (t_{thn}) and off (t_{the}) during the session of correction the trajectory of the spacecraft. An algorithm for choosing the parameters of the correction session is based on searching for acceptable values of the time instant t^*.

When searching for each fixed acceptable time t^*, the calculations are performed for the ballistic lifetime of the spacecraft and the light and shade situation onboard the spacecraft (during indicated lifetime) for a finite set $M\{V_{ch}\}$ of values of the increment of the characteristic velocity V_{ch}. Set $M\{V_{ch}\}$ is considered to be a set of all possible separate points (q pieces) that belong to a given segment $[V_{chmin}, V_{chmax}]$ of the number axis and is determined by given step $h_{Vch} > 0$ as follows:

$$M\{V_{ch}\} = \{V_{ch\,min}, V_{ch\,min} + h_{Vch}, \\ V_{ch\,min} + 2h_{Vch}, \ldots, V_{ch\,min} + qh_{Vch}\}. \tag{5}$$

In this case, it should be remembered that the quantity V_{ch} can take both positive and negative values and, hence, the values $V_{ch\,min}$ and $V_{ch\,max}$ can also be positive or negative values.

In connection with this, the solution to the problem of choosing the parameters of forthcoming correction can be reduced to solving the partial problem of choosing the correction parameters at which the increment of characteristic velocity V_{ch} is fixed value from set (5). In this case, the vector $\mathbf{e}$ is calculated from the fact that it is directed along the spacecraft velocity vector at the time instant t^*, assuming the passive spacecraft flight in orbit of the Earth's artificial satellite.

Problem of Selecting the Correction Parameters

$t_0, x_0, y_0, z_0, V_{x0}, V_{y0}, V_{z0}, s, S_d$ are the initial conditions of the spacecraft motion in the J2000 CS; m_0 is the value of the spacecraft mass m at time instant t_{the} of PS switching; V_{ch} is the increment of characteristic velocity as a result of the PS operation; t^* is the middle of the interval of the continuous PS operation in the correction session; $h_{\pi S}$ is acceptable flight altitude below the spacecraft; h_{tS} is a test step for the spacecraft lifetime (the fulfillment of condition (2)); and t_g is the time instant until which the verification of the ballistic spacecraft lifetime is implemented (in this case, it is taken to be $t_{1\,min} = t_{the}$ and $t_{1\,max} = t_g$).

Set: t_{thn}, Δt_{th} is the time instant when the PS is switched on to implement the correction of spacecraft motion and the duration of its operation;

$\mathbf{e}_{th}$ is unit vector (in the J2000 CS) of the direction of the PS thrust when correcting the spacecraft motion;

$\{t_{thn}, \mathbf{r}(t_{thn}, \mathbf{V}(t_{thn})\}$ are the kinematic parameters of the spacecraft motion in the J2000 CS at the time instant of finishing the PS operation;

$m_e = m(t_{the})$ is the spacecraft mass at the time instant when PS operation is finished, $m_e = m_0 - P/I_{sp}/g_0 \Delta t_{th}$ (only spacecraft mass losses are taken into account because of fuel consumption when correcting);

t_{le} is time of the ballistic lifetime of the operational spacecraft orbit (in this case, if the ballistic spacecraft lifetime is provided up to the time instant t_g, then it is accepted that $t_{le} = t_g$);

t_{shnj}, t_{shej}, t_{ashnj}, t_{ashej}, $K_{T\max_j}$ is the ordered (over the time of the beginning of shading time segments, $t_{shn1} < t_{shn2} < \ldots < t_{thnN}$) is a sequence of five numbers, each of which characterizes (after corrections): the beginning and end of the shading time segment ($[t_{shnj}, t_{shej}]$), the beginning and end of the segment of total solar eclipse ($[t_{ashnj}, t_{ashej}]$) and the degree of spacecraft shading (the coefficient $K_{T\max_j}$) on the *j*th shading-time segment (all of the considered segments belong to the interval $[t_{the}, t_{le}]$);

N is the number of five numbers indicated above, i.e., shading intervals, that belong to the interval $[t_{the}, t_{le}]$;

and an array of the following parameters of the osculating at the time instant t_{the} spacecraft orbit after correction in the J2000 CS, where h_π is altitude of pericenter above the Earth's surface, h_α is apocenter altitude above the Earth's surface, ω is argument of latitude pericenter, i is inclination, Ω is longitude of the ascending node, P_o is orbit period, t_π is the time instant when the pericenter of the orbit is passed by the spacecraft in the previous orbit, and t_Ω is the time instant when the beginning of the current flight orbit is passed by the spacecraft.

Here and below, it is accepted that, when calculating the altitudes of the perigee and apogee of the spacecraft orbit, because the Earth's shape is considered to be a sphere with an average radius of R_E = 6378.2 km; the values ω and Ω take the values from the half-interval $[0, 2\pi)$, and the value i is taken from the interval $[0, \pi]$. The number of the orbit is riced by one at the time instant when the spacecraft passes the ascending node, i.e., when the spacecraft crosses the reference plane of the J2000 CS and the applicate changes its sign from negative to positive.

5. SCHEME OF CORRECTING SPACECRAFT TRAJECTORY

Over time, when refining the parameters of spacecraft motion in the operational orbit formed after launching the spacecraft in November 2011, it became necessary to correct the spacecraft's trajectory in 2013. At the initial (before correction) spacecraft trajectory, its ballistic lifetime was restricted by the time instant that occurred at the end of 2013 to the beginning of 2014. This time instant was defined as the time when the spacecraft is first found at the altitude less than 400 km above the spherical Earth's surface. Moreover, at the indicated trajectory of spacecraft flight in the beginning of 2013 the spacecraft set into the Earth's shadow occurs, which essentially in the duration exceeds the maximum acceptable set (about 2 h) and is approximately 5.7 h.

Since the middle of November 2011, corrections were made on the future orbit of the spacecraft that included refining the initial conditions of the spacecraft's motion according the trajectory measurements and the TM data based on the laborious calculations of the parameters. For the target orbit (after corrections), we considered an orbit for which the time instant t_l occurs no earlier than in the middle of 2018 (at the expiration of 7 years after spacecraft launching into the operational orbit), and the spacecraft set into half-shadow does not exceed 2 h in duration by more than 10 min for 5 years after launching the spacecraft into the operational orbit of the Earth's artificial satellite. In this case, indicated conditions should be fulfilled taking into account errors of initial conditions of the spacecraft motion and the coefficient *Sd* of solar pressure, possible errors in the orientation and the value of PS thrust when implementing each of its switching. Preliminary calculations showed that the effect of errors in the thrust orientation on the further motion of the spacecraft CM is negligible compared with the influence of errors in the value of thrust, the current knowledge of the initial conditions, and the prediction of the value of coefficient *Sd*. All subsequent calculations of the correction parameters were performed taking into account the value of limit error of the PS thrust, which is equivalent to the relative error in the implementation of the increment of characteristic velocity V_{ch} equal to 9% of the value of increment and was previously agreed upon with the Main Operational Control Group (MOCK). In this case, there was a solution to the problem of the calculating correction parameters, which provides the above requirements for the spacecraft trajectory after correction for three values of the coefficient of light pressure, i.e., (1) *Sd* equal to the current (before correction) value *Sd0*, (2) $Sd = 0$, and (3) $Sd = 2Sd0$.

When solving the correction problems, the correction parameters are the time instants when the PS is switched on. Problems when the PS is switched on once and twice are considered. In the case when the PS is switched on twice, it should be possible to refine the spacecraft trajectory parameters before the second PS switching according the trajectory measurements and the TM data. Calculations have shown that this requirement is satisfied when the time instant at which the PS is switched on are spaced by no less than approximately the period of the satellite orbit with the practically possible intensity of the trajectory measurements. The possible direction of the PS thrust is selected to be almost uniquely based on the condition of its parallelism to the spacecraft velocity vector in view of restriction (4) by the γ angle.

The rejection of the correction with one PS switching occurs when the absolute value of the increment of characteristic velocity is so high that, after corrections, the trajectory is implemented with unacceptable errors.

The correction problem is a difficult mathematical programming problem, the functional of which is the sum of the absolute values of the increments of the characteristic velocities when the PS is switched on while making corrections, the value of which is proportional to the consumption of the working body for implementing the target correction. Searching for its solution is performed with human participation using the developed algorithms to solve the above basic problems for an orientation analysis and choosing the correction parameters.

When searching for the scheme of making correction to the *Spektr-R* spacecraft, schemes in which the correction sessions, as well as preliminary and final operations, are performed within visible zones for at least one of the two ground stations (in Medvezhyi Ozera and Ussuriisk), are preferable.

In the period from November 2011 to January 2012, at the ballistic center of the Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, the solution to more than 700 of the indicated problems of mathematical programming were performed in order to choose the parameters of spacecraft motion correction in 2012. Corresponding information for the forthcoming correction of spacecraft motion trajectory and the epy scheme for its implementation was presented by the Main Operational Group for Spacecraft Flight Control (MOCG). The indicated scheme has been designed in view of the need to implement a PS burn (a series of technological operations with the PS switching) before the first PS switching in order to implement the targeted correction of the operational spacecraft orbit. In this case, it was taken into account that the PS burn leads to an increment of the characteristic spacecraft velocity within 2 cm/s. As a result, the following decisions were made:

(1) Corrections are performed in order to provide (when maintaining the model of forces acting on the spacecraft) a ballistic spacecraft lifetime until the middle of 2018 (the spacecraft altitude above the Earth's surface is not less than 640 km); the absence of continuous intervals of shadow on the spacecraft from the Earth with an unacceptable shading coefficient for a duration of more than 2.2 h before the beginning of 2017; and the conservation (for carrying out effective researches) of the evolution of spacecraft orbit, which is achieved upon small variations of the spacecraft orbit parameters.

(2) Correction is implemented according the scheme proposed by KIAM: burn + first pulse on February 21, 2012; second pulse on March 1, 2012. Reserve versions: (1) burn and the first correction pulse on February 21, 2012; the second correction pulse on March 10, 2012; (2) burn and the first correction pulse on March 1, 2012; the second correction pulse on March 10, 2012. In all cases, the ballistic parameters necessary for the implementation of the second correction pulse are calculated using the trajectory measurements after implementing the first correction pulse taking into account the possible implementation of the second pulse with error with respect to modulus not exceeding 9% of its absolute value. All versions provide the conditions for correcting the operational spacecraft orbit. The first pulse is about 1.49 m/s. The burn imparts a total pulse of about 0.01 m/s to the spacecraft. The second pulse is about 2 m/s. The beginning of PS operation for the pulse implementation occurs on February 21, 2012 at 21.00.00. Switching PS to implement the second pulse occurs in the region of the apocenter of the current orbit.

All necessary ballistic data for the real implementation of sessions for correcting operational spacecraft orbit were calculated in accordance with the above scheme.

The total duration of the PS operation at burning was determined to be equal to 2 s. The time instants of PS switching were taken to be no more than a few minutes before the PS was switched on in the first session of the target correction of the spacecraft orbit. In ballistic calculations, the burn was simulated as one interval of the PS operation for 2 s. In this case, the average time t^* of the PS operational interval coincided with the middle of the interval between the beginning of the first and the end of the last from expected PS switching when implementing the burn and the direction of the PS thrust coincided with the direction of the thrust in the first session of the target correction.

For simulating burn, t^* = 21.02.2012 at 20.56.47,1 was agreed with MOCG. As a result of burn, the estimated value of the absolute increment of characteristic velocity is about 0.01 m/s.

In the first session of the target correction of the operational satellite orbit, PS should be switched on February 21, 2012 at 21.00.00,0 and operate during 300 s. At the cost of this PS operation, the calculated value of the absolute increment of the characteristic spacecraft velocity is about 1.75 m/s. In the second session of the target correction of the operational satellite orbit, PS should be switched on March 1, 2012 at 14.45.00,0 and operate during 332 s. At the cost of this PS operation, the calculated value of the absolute increment of the characteristic spacecraft velocity is about 1.86 m/s. The above-mentioned burn and two sessions of the target spacecraft correction were performed.

6. ANALYSIS OF THE RESULTS OF SPACECRAFT TRAJECTORY CORRECTION

An analysis of the results of executing the target correction of the *Spektr-R* spacecraft is performed by comparing the characteristics of sequences of three trajectories of its passive flight, i.e., before PS burning, refined after corrections, and current (refined on November 20, 2013) trajectory. The indicated trajec-

Table 1. Ballistic lifetime and intervals of spacecraft shading

Number of trajectory	0	1	2
t_{she}	22.XII.2013 08.04.29	14.I.2020 21.45.22	>18.VII.2021 12.00.00
t_{shn}	8.I.2013 22.04.19	11.I.2018 12.29.27	21.I.2017 11.36.36
δt_{sh}, hours	20.492	22.129	24.062
t_{ashn}	8.I.2013 23.05.35	11.I.2018 13.50.06	21.I.2017 13.20.55
δt_{ash}, hours	13.062	12.118	11.688

tories are assigned to corresponding numbers of 0, 1, and 2.

Table 1 shows the calculated characteristics of ballistic lifetime and the first unacceptable spacecraft shading during flight with the coefficient for each of the three trajectories. The following additional designations are used in the table, except for previously presented designations: $\delta t_{sh} = t_{she} - t_{shn}$ is the duration of the shading interval and $\delta t_{ash} = t_{ashe} - t_{ashn}$ is the duration of the total solar eclipse.

The data in Table 1 show that the assigned problems of correcting the operational orbit are successfully solved with regard to the support (when conserving the model of forces acting on the satellite) of long-term ballistic spacecraft lifetime and acceptable light and shade situation onboard the spacecraft to about the beginning of 2017.

In order to get information on the difference between trajectories 0, 1, and 2, for each of them, at certain intervals of the spacecraft flight, the parameters of osculating orbits were calculated in the sequence (by orbits) of time instants in order to reach the minimum distance of the spacecraft from the Earth's CM. The following previously introduced parameters are considered to be the parameters of the osculating orbits: h_π, h_α, ω, i, Ω, P_o and two parameters that are interesting from the point of view of implementing the scientific program of the *Spektr-R* spacecraft, namely, the right ascension α_e and the declination δ_e of apocenter in the rectangular right ecliptic coordinate system (ECS), the beginning of which coincides with the beginning of the J2000 CS. The directions of the abscissa axes of these CS coincide. The applicate axis of ECS is orthogonal to the plane of the ecliptic and directed towards the Earth's North Pole.

The calculation results are shown in Figs. 1–16 in the form of graphs with broken lines. The number axis is considered to be the abscissa axis. Abscissa values of the vertices of broken lines belong to a finite set $jk(D_1, D_2)$ of natural numbers. Elements of this set are serial numbers of points in the course of the satellite flight from the beginning of a date (D_1) up to the beginning of a date (D_2), on the kth trajectory with minimal (in orbit) distance from the Earth's CM. The ordinate axis of the vertex of the broken line is equal to the calculated value of the parameter of the osculating orbit indicated in the figure. The number (k) of the trajectory is specified directly next to the broken lines. Instead of the designations h_π and h_α, the designations h_{min} and h_{max} are used, respectively.

Figures 1–8 show the dependences of the parameters of osculating orbits for trajectories 0 and 1 on the interval, the beginning of which is approached to the time instant of finishing the last session of correction, where D_1 = 04.03.2012 and D_2 = 22.12.2013. These values reflect the changes in the characteristics of the

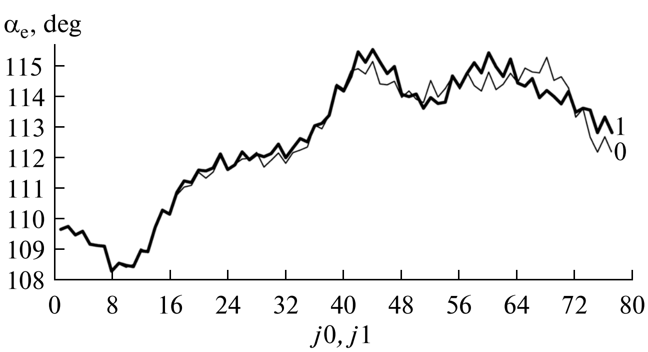

Fig. 1.

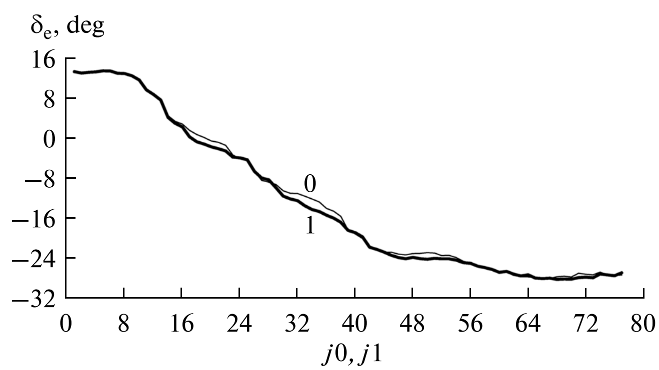

Fig. 2.

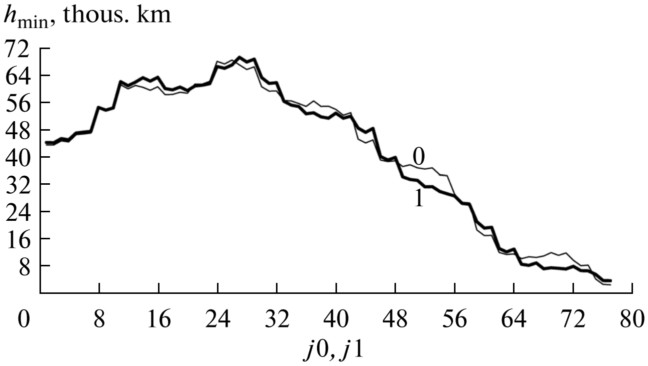

Fig. 3.

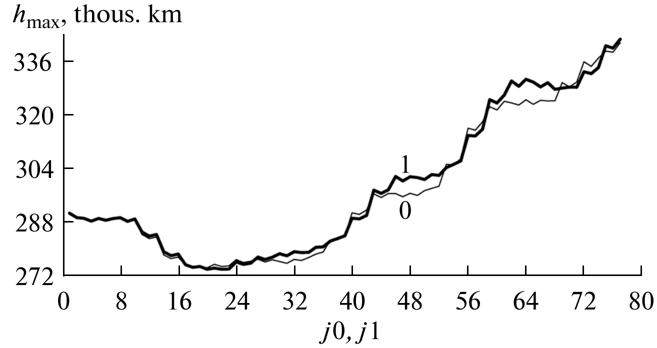

Fig. 4.

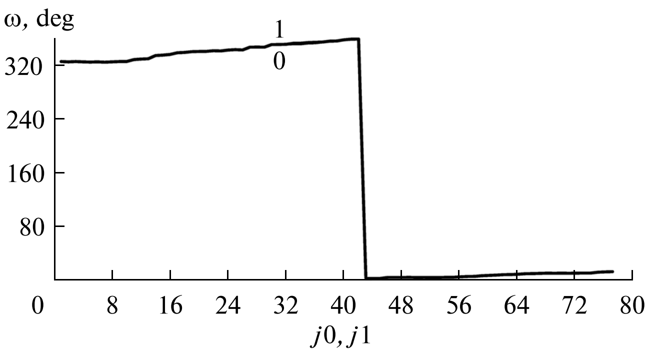

Fig. 5.

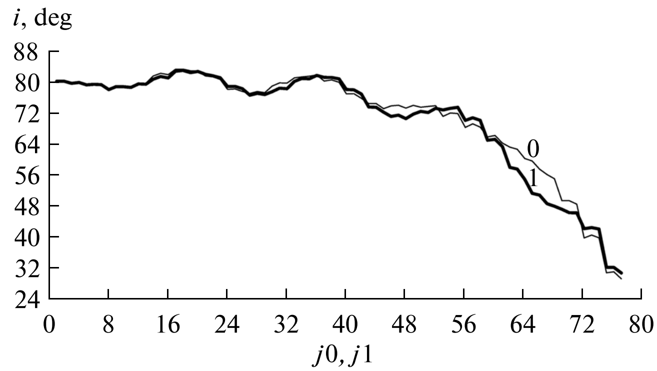

Fig. 6.

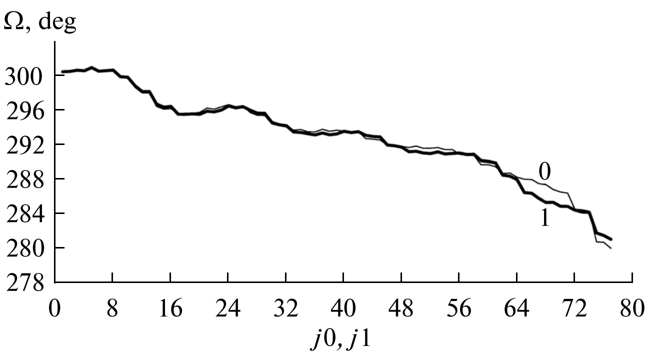

Fig. 7.

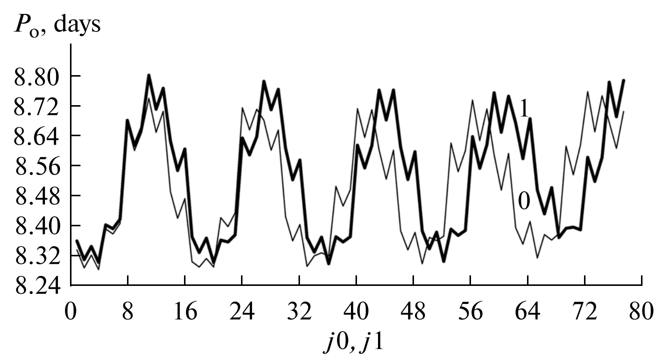

Fig. 8.

operation spacecraft orbit as a result of burning and correcting the trajectory of its motion.

Figures 9–16 show the dependences of the same parameters of osculating orbits for trajectories 1 and 2 as in Figs. 1–8 over an interval of about 5 years. The beginning of this interval is more than 1.5 year away from the time instant of the completion of the last session of correction and coincides with the end of the interval in Figs. 1–8, where $D_1 = 22.12.2013$ and $D_2 = 20.07.2018$.

Tables 2 and 3 set up a correspondence between numbers of jk points on the kth trajectory, where the spacecraft reaches minimum (in orbit) distance from the Earth's CM and calendar time.

The dependences shown in Figs. 9–16 allow us to make a qualitative estimate of the model of the *Spektr-R* spacecraft motion used in ballistic calculations. The motion model is adequate for real satellite motion from the point of view of scheduling the flight control of the spacecraft over a few years.

Fig. 9.

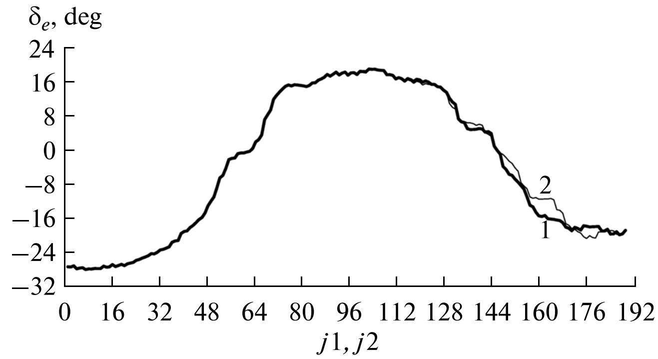

Fig. 10.

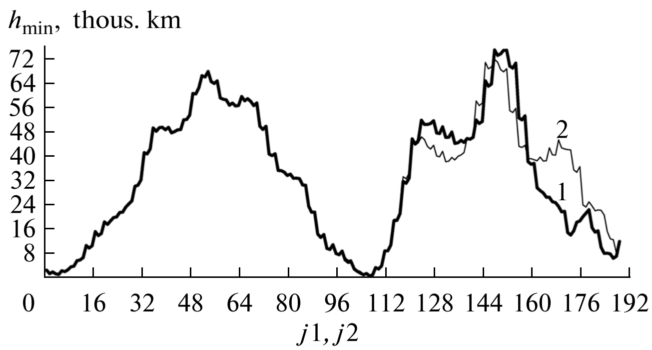

Fig. 11.

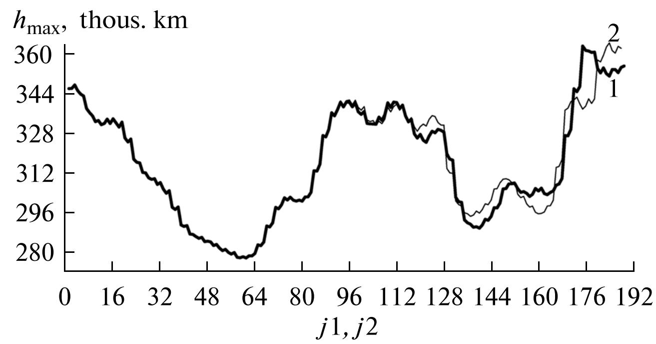

Fig. 12.

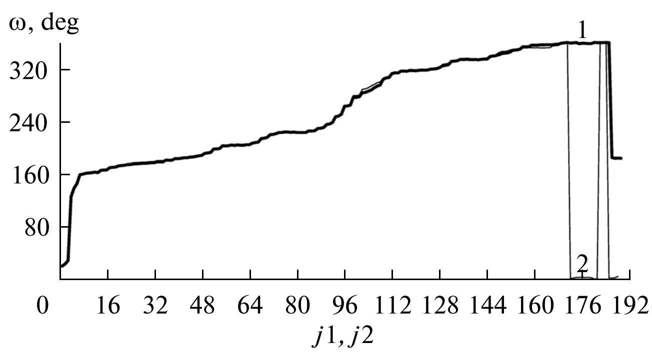

Fig. 13.

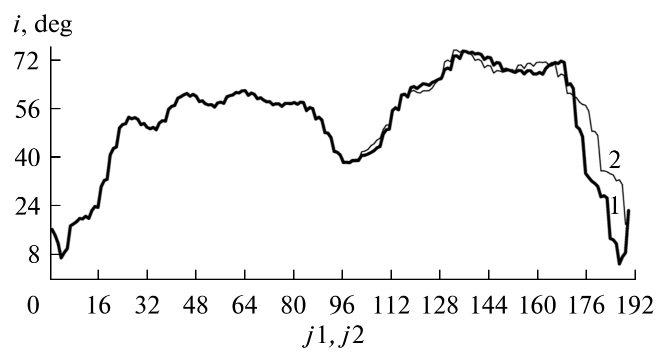

Fig. 14.

CONCLUSIONS

The trajectory of the spacecraft's flight is corrected in accordance with the proposed basic scheme. The calculated time of switching on the PS in the first correction session is on February 21, 2012 at 21.00.00,0 and in the second correction session on March 1, 2012 at 14.45.00,0. In this case, in agreement with the technical administration of the spacecraft flight, the burning of the spacecraft orbit in ballistic calculations was simulated by switching on the PS for 2 s of continuous operation at the time instant of February 21, 2012 at 20.56.46,1. The calculated increments of the characteristic spacecraft velocity during PS burning and the implementation of the first and second correction sessions were 0.01, 1.75, and 1.86 m/s, respectively.

After finishing the correction, subsequent calculations showed (see Tables 1–3 and Figs. 1–16) that the correction was implemented successfully, i.e., the basic requirements for flight trajectory after corrections are performed, when keeping the mathematical

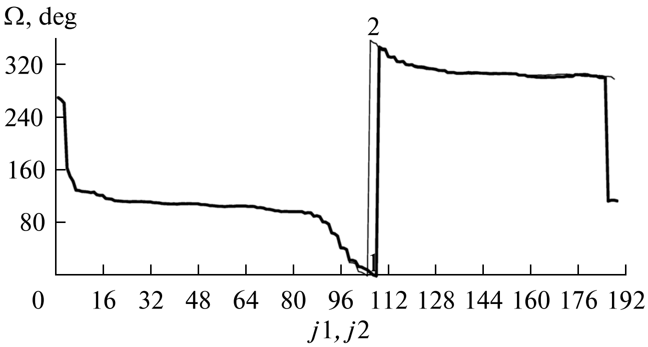

Fig. 15.

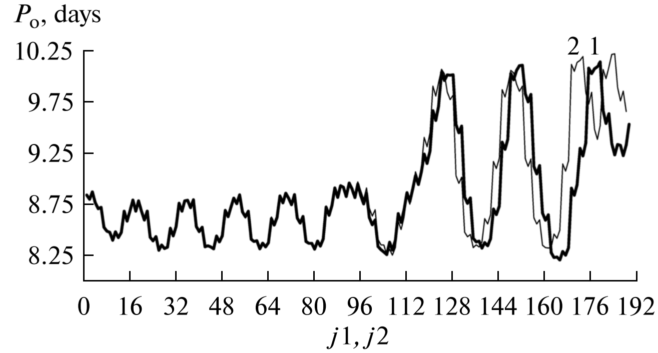

Fig. 16.

Tables 2. Instant times for approaching minimum distance for trajectories with numbers 0 and 1 from March 4, 2012 to December 22, 2013

*j*0	Date	Time	*j*1	Date	Time
1	5.III.2012	21.07.58,844	1	5.III.2012	21.45.26,826
6	16.IV.2012	09.09.17,801	6	16.IV.2012	12.00.28,656
11	29.V.2012	11.32.48,964	11	29.V.2012	15.38.36,449
16	11.VII.2012	18.48.31,059	16	12.VII.2012	11.32.12,122
21	22.VIII.2012	13.37.36,110	21	23.VIII.2012	12.48.21,469
26	4.X.2012	12.04.50,820	26	5.X.2012	01.10.09,638
31	16.XI.2012	17.07.19,668	31	17.XI.2012	22.07.01,070
36	28.XII.2012	11.27.19,353	36	30.XII.2012	02.55.54,760
41	9.II.2013	15.06.40,917	41	10.II.2013	13.51.14,225
46	24.III.2013	19.54.00,465	46	26.III.2013	08.38.35,630
51	5.V.2013	10.34.28,336	51	7.V.2013	14.28.15,897
56	17.VI.2013	08.49.22,710	56	18.VI.2013	14.27.05,944
61	30.VII.2013	18.28.47,007	61	1.VIII.2013	05.42.36,166
66	10.IX.2013	16.30.21,606	66	13.IX.2013	08.31.48,257
71	22.X.2013	22.04.53,124	71	25.X.2013	02.36.23,591
76	5.XII.2013	05.09.38,125	76	6.XII.2013	22.39.45,207
77	13.XII.2013	19.26.05,325	77	15.XII.2013	14.55.32,645

Tables 3. Instant times for approaching minimum distance for trajectories with numbers 1 and 2 from December 22, 2013 to July 20, 2018

$j1$	Date	Time	$j2$	Date	Time
1	24.XII.2013	09.40.44,974	1	24.XII.2013	08.46.37,707
6	5.II.2014	16.31.07,614	6	5.II.2014	14.52.32,823
11	19.III.2014	14.26.31,051	11	19.III.2014	12.23.07,511
16	1.V.2014	01.37.26,802	16	1.V.2014	00.31.57,852
21	13.VI.2014	18.09.55,794	21	13.VI.2014	16.57.56,549
26	26.VII.2014	10.55.44,955	26	26.VII.2014	08.31.20,277
31	6.IX.2014	10.27.10,341	31	6.IX.2014	08.39.45,627
36	20.X.2014	00.57.55,052	36	20.X.2014	00.15.07,420
41	2.XII.2014	09.15.50,866	41	2.XII.2014	07.24.33,697
46	13.I.2015	07.59.44,944	46	13.I.2015	05.52.52,336
51	25.II.2015	09.38.59,431	51	25.II.2015	08.50.04,079
56	10.IV.2015	16.33.56,866	56	10.IV.2015	15.12.47,832
61	23.V.2015	00.42.12,985	61	22.V.2015	22.04.23,813
66	4.VII.2015	08.08.39,040	66	4.VII.2015	06.43.53,458
71	17.VIII.2015	21.14.04,220	71	17.VIII.2015	20.49.11,524
76	30.IX.2015	13.56.42,976	76	30.IX.2015	12.44.26,176
81	11.XI.2015	07.25.53,389	81	11.XI.2015	05.53.29,067
86	24.XII.2015	04.32.56,136	86	24.XII.2015	04.12.21,949
91	7.II.2016	04.22.14,270	91	7.II.2016	04.54.02,232
96	22.III.2016	19.57.40,505	96	22.III.2016	22.53.45,266
101	5.V.2016	05.52.07,501	101	5.V.2016	16.53.31,000
106	16.VI.2016	00.51.07,116	106	16.VI.2016	16.34.14,638
111	28.VII.2016	23.55.49,689	111	29.VII.2016	07.33.36,474
116	11.IX.2016	14.22.31,448	116	11.IX.2016	20.44.25,137
121	27.X.2016	14.51.41,697	121	28.X.2016	11.24.35,857
126	15.XII.2016	16.23.21,437	126	17.XII.2016	04.02.31,477
131	2.II.2017	21.38.29,864	131	2.II.2017	20.53.04,145
136	19.III.2017	05.20.40,978	136	17.III.2017	12.37.39,221
141	29.IV.2017	22.09.38,920	141	28.IV.2017	19.52.11,571
146	13.VI.2017	12.56.31,166	146	14.VI.2017	18.18.02,023
151	1.VIII.2017	23.32.57,999	151	3.VIII.2017	15.04.01,096
156	20.IX.2017	13.23.42,452	156	19.IX.2017	19.24.30,134
161	4.XI.2017	00.24.30,866	161	1.XI.2017	00.37.37,458
166	15.XII.2017	08.07.26,097	166	13.XII.2017	15.19.48,370
171	27.I.2018	20.22.18,898	171	31.I.2018	08.38.48,469
176	17.III.2018	00.16.24,622	176	22.III.2018	10.50.15,360
181	5.V.2018	10.26.18,450	181	9.V.2018	19.32.16,713
186	21.VI.2018	01.11.45,195	186	28.VI.2018	22.10.03,073
189	18.VII.2018	22.45.19,348			

model of the spacecraft motion. Moreover, the calculated data presented in this paper testify to the adequacy of the model for the ballistic motion of *Spektr-R* spacecraft to correct its motion.

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