ISSN 0010-9525, Cosmic Research, 2014, Vol. 52, No. 5, pp. 332–341. © Pleiades Publishing, Ltd., 2014.
Original Russian Text © N.S. Kardashev, B.B. Kreisman, A.V. Pogodin, Yu.N. Ponomarev, E.N. Filippova, A.I. Sheikhet, 2014, published in Kosmicheskie Issledovaniya, 2014, Vol. 52, No. 5, pp. 366–375.

Orbit Design for the *Spektr-R* Spacecraft of the Ground–Space Interferometer

N. S. Kardashev[a], B. B. Kreisman[a], A. V. Pogodin[b], Yu. N. Ponomarev[a], E. N. Filippova[b], and A. I. Sheikhet[b]

[a] *Astrospace Center, Lebedev Physical Institute, Russian Academy of Sciences, Moscow, Russia*
e-mail: yupon@asc.rssi.ru
[b] *Lavochkin NPO (Science and Production Corporation), Khimki, Moscow oblast, Russia*

Received December 16, 2013

Abstract—In order to carry out tasks of the RadioAstron mission, a high-apogee orbit was designed. On average, the period of its satellite's orbit around the Earth is 8.5 days with evolution due to gravitational perturbations produced by the Moon and the Sun. The perigee and apogee of this orbit vary within the limits 7500–70 000 km and 270 000–333 000 km, respectively. The basic evolution of the orbit represents a rotation of its plane around the line of apsides. Over 3 years, the plane normal to the orbit draws on the celestial sphere an oval with a semi-major axis of about 150° and semi-minor axis of about 45°.

DOI: 10.1134/S0010952514050050

1. INTRODUCTION

RadioAstron is a Russian project that includes some collaborators from abroad. The space radio telescope (SRT) of the RadioAstron mission whose antenna has a diameter of 10 m was injected on July 18, 2011 into a high-apogee orbit from the Baikonur space launch facility using carrier launcher *Zenit* and boosting block *Fregat-SB*. Joint operation with ground-based radio telescopes (GRT) provides for a possibility to create a ground–space radio interferometer with essentially longer base as compared to ground-surface interferometers, which allows one to get much higher angular resolution (down to 10 microseconds of arc) than that of ground-based instruments. One can find basic technical characteristics of the SRT on the mission website.[1]

Originally, the launch of SRT of the RadioAstron mission was planned on 1994, and an orbit with the following initial parameters was selected:

Perigee height, km	4000
Apogee height, km	768000
Inclination, deg	51.5
Perigee argument, deg	300.0
Longitude of ascending node, deg	190.0
Period of revolution, h	28

The selection of an orbit with the above parameters was determined by the scientific tasks of the mission (maximum possible survey of the celestial sphere over the mission period and optimization of observation sessions taking into account the possibility of constructing the Earth–SRT interferometer).

This orbit has a rather weak evolution. For example, for three years the perigee height increases from 4000 up to 11297 km, while the apogee height decreases from an initial height of 76 800 down to 69491 km. Its inclination increases from 51.5° to 55.5°, while the ascending node longitude decreases from 190° to 156.9°.

The perigee argument value 300° was determined by the fact that the mission control stations (MCS) are located on the territory of Russia. Therefore, to ensure good conditions of visibility, when making commands and transmitting service telemetry, one must have the perigee argument in the fourth quadrant, i.e., $270 \leq \omega_{ecl} \leq 360°$. Taking into account that the ecliptic plane is tilted to the Earth's equator plane by an angle of 23.5° and that the minimum elevation angle for good spacecraft visibility is 7°, the perigee argument ω was chosen to be equal to 300° with respect to the equatorial plane ascending node. The choice of revolution period equal to 28 h is explained by the fact that, in this case, the spacecraft makes exactly six orbits in a week. This is very convenient for planning and executing spacecraft control operations.

For objective reasons, the spacecraft launch was postponed several times. In February 1997, a Japanese telescope (VSOP project) was launched into orbit with an apogee of 20 000 km. It has carried out long observations with bases up to 30 000 km before the *Spektr-R* spacecraft was launched. In order to take the following step (to improve considerably the angular resolution

[1] http://www.asc.rssi.ru/radioastron/index.html

and quality of images), it is necessary to use a high-apogee orbit that strongly evolves under the action of the Moon and the Sun.

In 2002, the decision was made to inject the spacecraft of the RadioAstron mission into a higher orbit. The planned apogee height of the *Spektr-R* spacecraft was elevated from 80000 to 350000 km, which required works on choosing the new high-apogee orbit with an apogee radius of 300000–350000 km.

In addition, in 2004 the decision was made to design the new *Spektr-R* spacecraft based on the *Navigator* platform. This change in spacecraft resulted in a change in the system of attitude and stabilization control of the space radio telescope, as well as of the system of spacecraft orbit correction, which required new a priori estimations of the accuracy of predicting orbit evolution taking into account new estimates of the time intervals between the unloading of the angular momentum accumulated due to direct and reflected solar radiation, as well as due to gravitational moments on perigee segments of the orbit.

To carry out these tasks, the mission requires an orbit with a strong evolution of the orbit plane, which would allow one to make observations both with small and with long bases, comparable to the apogee height of ~330000 km. Some constraints on initial orbit parameters have a strong influence on evolution of the orbit plane. They are the initial perigee height and inclination (600 km and 51.4°, respectively), the orbit lifetime (no shorter than 9 years), duration of shadows, and so on. The evolution of the orbit is determined by changes in the instantaneous values of the ellipse parameters in the orbit plane and by changing the position of the orbit plane, i.e., the direction of a vector normal to the orbit plane, in space.

When a spacecraft moves along a high-apogee orbit with large eccentricity, main perturbations are produced by attraction of the Moon and the Sun, and by the nonsphericity of the Earth's gravity field. The significant influence of these factors was observed for the first time when analyzing the motion of the *Automatic Interplanetary Station (AIS)* launched to the Moon in 1959. After a rendezvous with the Moon, the *AIS* became a satellite of the Earth with a perigee height of 47000 km. In 11 orbits, the orbit's perigee height decreased substantially and the *AIS* ended its existence [1].

When studying the evolution of these orbits in the general case, one must investigate a broad area of possible values of five orbital parameters. For this investigation, one should use numerical solutions of the system of differential equations of spacecraft motion with the fullest possible model of perturbations, but this requires bulky computer calculations and laborious subsequent analysis. Therefore, at the initial stage of choosing the orbit, which should ensure the solution of scientific tasks, approximate methods were used that allowed for the qualitative regularities of orbit evolution to be studied and quantitative estimates to be obtained for this evolution over a long time interval. The knowledge of qualitative regularities allows one to substantially narrow the region of possible values of the initial orbit parameters. The additional contraction of the region of possible values of these parameters is ensured by the constraints imposed by power characteristics of the carrier launcher and booster block, by time of ballistic existence, and by geography of a launching site and ground-based control points.

The use of the analytical model of spacecraft motion that takes into account basic perturbing factors (oblateness of the Earth, gravitational influence of the Moon and the Sun) makes it possible to predict the motion of the spacecraft over long time intervals and to promptly analyze the obtained orbits without sophisticated numerical calculations.

The qualitative analysis of orbit evolution under the action of an external perturbing body was performed by M.L. Lidov as early as in 1961 [2]. In order to analyze the motion of the spacecraft in long intervals, he used a doubly averaged restricted circular three-body problem, which is integrable by quadratures.

In paper [3], M.A. Vashkov'yak and M.L. Lidov described two classes of high-apogee orbits whose longitude of ascending orbit, the inclination of the orbit plane, and the pericenter argument were strongly variable for three years. More papers were published by these authors on the evolution of a high-apogee orbit due to the gravitational influence of the Moon and Sun. They pointed out the possibility of designing a high-apogee orbit with strong evolution for the SRT. Over the years, the date of launch of the space radio telescope was postponed several times. Therefore, methods of constructing and studying the evolution of the SRT orbit have been changing.

2. ELABORATION OF ORBIT QUALITY CRITERION

As a result of investigations made at ASC and Lavochkin NPO, the criterion of orbit quality was elaborated, which takes into account the basic requirements for the SRT orbit needed to solve the project's scientific tasks. Below, we present a formal description of this criterion and constraints.

In order to estimate the quality of weakly evolving orbits, it is sufficient to analyze the occupancy of the (uv) plane for most important objects on the interval of several satellite orbits.

For strongly evolving orbits the situation is much more complicated. For a space experiment lasting on order of 3–5 years, this orbit should make it possible to distribute this time to observe objects of the entire celestial sphere (or at least belonging to specified regions of the celestial sphere) and to obtain high-quality images of all such objects. It has been suggested that, as an estimate of suitability of an orbit for obser-

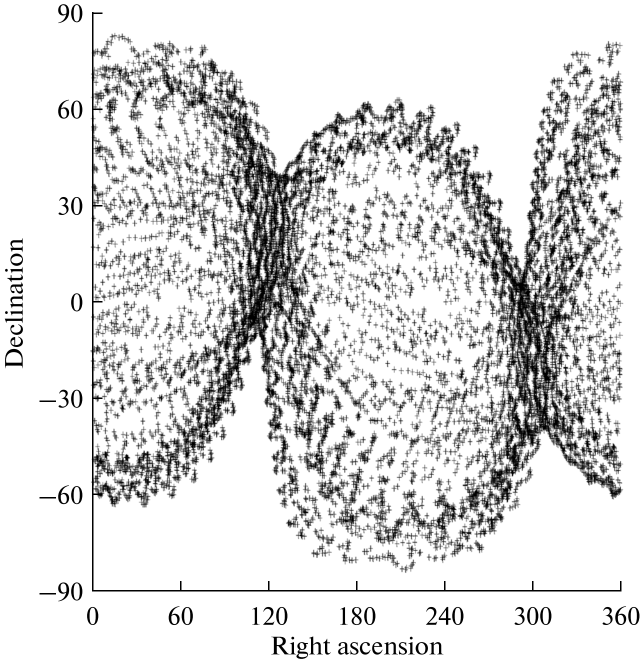

Fig. 1. Evolution of direction of the Earth–SRT vector onto the celestial sphere.

vation of some object of the celestial sphere, one should use the total duration of time intervals during which the Earth–SRT vector is directed to the vicinity of this object (Fig. 1). The usefulness of an orbit to obtain high-quality images of a given list of objects is described by a set of these estimates for every object of the list [4]. The efficiency criterion Φ developed for the RadioAstron project is used for the quantitative estimation of the effectiveness of the selected orbit.

In order to choose the best orbit that allows one to investigate the objects on the largest possible part of the celestial sphere for the longest possible time, it is partitioned into $N \times M$ pads of equal area in the galactic coordinate system as follows: longitude is divided in N equal intervals, $360/N$ degrees each and latitude is divided in M unequal intervals $[Q(m-1), Q(m)]$, the boundaries of which $Q(m)$ are determined from the condition of equal areas according to the formula

$$Q(0) = 90, Q(m) = \arcsin(1 - 2m/M), m = 1, 2, \ldots, M.$$

Pads S on the sphere are identified by pairs (n, m), where n is the number of half-interval to which longitude belongs ($n = 1, 2, \ldots, N$), and m is the number of half-interval to which latitude belongs ($m = 1, 2, \ldots, M$).

For every pad $S(n, m)$, we sequentially determine the time periods when it is possible to organize an observation session for an object located in the center of the pad. Then, durations $D(n, m, k)$ of time periods when the base projection onto (uv) plane lies within the limits $[p(k-1), p(k)]$, $k = 1, 2, \ldots, K$ are calculated and accumulated. Roughly, $p(0) = 0$, $p(1) = 15000$, $p(2) = 50000$, $p(3) = 100000$, $p(4) = 150000$, $p(5) = 200000$, $p(6) = 250000$, $p(7) = 450000$ km.

If the total (over all possible sessions) duration of these intervals is less than $T_{\min}$, then time $D(n, m, k)$ is considered to be zero. For every k, the number $\mathbf{l}(k)$ of pads for which corrected time $D(n, m, k)$ is not equal to zero is determined. Criterion Φ of orbit quality in the RadioAstron project is calculated by the formula

$$\Phi = \sum_{k=1}^{7} q(k) l(k),$$

where $q(k)$, $k = 1, 2, \ldots, K$ are weight coefficients.

It was assumed that $N = 50$, $M = 20$, $T_{\min} = 10$ min, $q(1) = 8$, $q(2) = 5.7$, $q(3) = 4$, $q(4) = 2.9$, $q(5) = 2$, $q(6) = 1.4$, and $q(7) = 1$. If the orbit lifetime is less than 9 years, we assume $\Phi = 0$. Matrix $D(n, m, k)$ and vector $\mathbf{l}(k)$ are used in additional analysis of the orbit quality. A quarter of the celestial sphere should be always observable.

When searching for optimal values of the working orbit initial parameters, it is necessary to maximize the value of functional Φ, which is a criterion of orbit quality and is shortly defined above. This optimization is possible every month. For example, in calculations made for the dates of launch October 7, 2000 and March 15, 2006 maximum values of the functional were $\Phi = 13998$ and $\Phi = 16500$, respectively.

3. CONSTRAINTS ON SRT LAUNCH AND ORBIT CORRECTION

When carrying out investigations in order to choose initial parameters of the working orbit of *Spektr-R*, the orbit elements can be divided in two groups, i.e., constants and variables.

The constants are as follows: orbit inclination is determined by allowed azimuth of shooting at the Baikonur space launch facility and equals $i = 51.4°$; initial height of the pericenter is determined by power capability of the booster block and is assumed to be $H_\pi = 600$ km.

The following quantities are considered to be variables: pericenter argument ω within the limits 280°–370°, ascending node longitude Ω within the limits 0°–360°, and apocenter height H_a in the range 300000–350000 km.

Preliminary estimates of accuracy with which initial parameters of the spacecraft working orbit are formed have shown that the spacecraft is injected into working orbit with errors (3σ) that do not exceed the following values: for apogee height, $\Delta H_a = \pm 2000$ km; for perigee height, $\Delta H_\pi = \pm 8$ km; for inclination, $\Delta i = \pm 3$ arcmin; for the pericenter argument, $\Delta\omega = \pm 10$ arcmin; and, for the longitude of the ascending node, $\Delta\Omega = \pm 8$ arcmin.

When choosing the initial parameters of the spacecraft working orbit, one should take into account the following constraints:

—the duration of scientific experiments is no shorter than three years; the time of ballistic existence is no less than 9 years;

—the duration of shadowing (shadow plus penumbra) on an orbit should not exceed 2 h (with allowance made for possible correction of phasing);

—the time of ballistic existence and duration of shadowing per orbit must be ensured taking into account the scatter of initial parameters of the working orbit at injection. In this case, the total increment of characteristic velocity of all corrections should not exceed 60 m/s.

The initial height of the pericenter of the spacecraft's working orbit determined by the power capability of a carrier launcher can lie in the range of 400–1000 km (after forming the program of spacecraft launch the pericenter height was fixed at 600 km). This imposes additional constraints on the permissible region of the initial values of perigee argument ω and longitude Ω of the ascending node of the spacecraft working orbit.

When an object on the celestial sphere is observed from the spacecraft, all constraints on orientation of onboard and ground-based facilities should be fulfilled, and communication with at least one tracking station should be established. The duration of an observation session without reorientation is no less than 1 h.

4. CALCULATIONS

4.1. First Stage (Restricted Three-Body Problem)

At the first stage, we have started studying the orbit evolution using the three-body problem. Since only one integral exists in the restricted three-body problem (the Jacobi integral), it is impossible to find the total set of solutions. In connection with this, periodic solutions are usually investigated. These investigations are based on the hypothesis by Poincare who suggested that, if there is a partial solution of the restricted problem, one always can find a periodic solution (may be with a very long period) that possesses the following property: at any t, its difference from the original partial solution can be as small as one wishes.

Algorithms and programs were developed to construct periodic solutions of the restricted three-body problem for planar and three-dimensional cases. Using this software, various families of periodic solutions were constructed [5]. It is demonstrated that even orbits with apogee lesser than 300000 km (i.e., formally located beyond the sphere of action of the Moon) can have intense motion of the line of apsides. The orbits with a period of 9 days are selected as most promising for use in RadioAstron projects.

The most interesting families of periodic solutions were constructed for the Earth–Moon system. It is shown that, in the four-dimensional phase space, if the distance from some orbit to any orbit of flying around the libration point L_2 is very small, then the apsidal line turn can reach several weeks. Among these orbits, one can choose an orbit with any preset turn of the apsidal line that passes far from the Moon; this can be used for space maneuvering.

Periodic solutions in which the motions around attracting bodies alternate with multiple flybys of libration points L_1 and (or) L_2 were also constructed. The periodic solution record in complexity combines four types of orbits and includes multiple flybys of libration points L_1 and L_2 of the Earth–Moon system. It is shown that, in solutions of this type, one is able to increase the number of flybys of the librations point unlimitedly and to obtain doubly asymptotic solutions. Examples of solutions with multiple commensurabilities were presented. A special report[2] was prepared based on the results of these investigations.

Using the estimates of orbits that explicitly take into account the quality of images obtained as a result of processing the space interferometer data, the most promising orbits were selected (allowing for the possibility of launching SRT by the *Proton* rocket and in the future by the *Zenit* rocket) for the RadioAstron project [6].

To construct (in the vicinity of periodic solution) a real trajectory of the RadioAstron project spacecraft, a program was developed for numerically solving the spacecraft's equations of motion, where the geopotential model GEM-T2 [7] was used. In order to take into account lunisolar perturbations, the model DE403/LE403 developed in NASA JPL was used [8].

The spacecraft equations of motion were integrated in the inertial coordinate system bound with the center of mass of the Earth and with the equatorial plane specified for the epoch J2000.0. The x axis is directed to the vernal equinox point γ. The z axis is aligned with the Earth's axis of rotation, while the y axis completes the right-hand triple. The same coordinate system was also used in all subsequent calculations.

4.2. Second Stage: Generalized Problem of Two Immobile Centers and Perturbations

When designing the orbit of the space radio telescope of the RadioAstron mission, it turned out that the generalized problem of two immobile centers yielded a more effective solution, since, unlike the three-body problem, it allows one to take into consid-

[2] Yu.N. Ponomarev, B.B. Kreisman, P.A. Tychina, and K.A. Kochetkov, "Alternative Working Orbits for Spacecraft of the RadioAstron Project: Investigation of the Possibility of Designing Orbits that Strongly Evolve under the Action of Lunisolar Perturbations", Scientific and Technical Report, Astrospace Center, Lebedev Physical Institute, Moscow, 1997.

eration that gravitational fields of the Earth and the Moon were not pointlike.

In this problem, a material point moves in the gravitational field produced by two attracting bodies immobile with respect to each other. This statement of the problem was first formulated by L. Euler, who also found its solution. In 1961, E.P. Aksenov, E.A. Grebennikov, and V.G. Demin [9] suggested using the Euler problem of two immobile centers to construct the theory of motion of spacecraft. To do this, they took the potential of two centers located at a fixed imaginary distance *2ic* from one another the basis of the problem. This was a certain generalization of the Euler problem of motion of a material point in a field of two immobile attracting centers.

When choosing initial parameters of the working orbit of the *Spektr-R* spacecraft we assume that the space radio telescope is a material point which moves in the noncentral field of attraction of the Earth and is subject to various perturbations from the Moon, Sun, light pressure, etc. Thus, the Euler orbit was used as a first approximation orbit for the analytical estimation of perturbations of the SRT orbit. Motion along this orbit is determined by the potential W of the generalized problem of two immobile attracting centers with masses $m/2(1 + i\sigma)$ and $m/2(1 - i\sigma)$, which are located at a fixed imaginary distance *2ic* from each other. The potential of the generalized problem of two immobile centers possesses axial symmetry, and it is a rather good approximation to the real gravitational potential of the Earth. The advantage of this intermediate potential is the fact that in this case differential equations of motion are exactly integrable by quadratures.

For the Earth's gravitational field, symmetric about the z axis and about the equatorial plane, the field potential can be considered a function of geocentric coordinates x, y, and z [10]

$$W = \frac{\mu}{2}\left[\frac{1 + i\sigma}{r_1} + \frac{1 - i\sigma}{r_2}\right],$$

where $i = \sqrt{-1}$, c and σ are real constants; $\mu = fm$; and

$$r_1 = \sqrt{x^2 + y^2 + [z - c(\sigma + i)]^2},$$

$$r_2 = \sqrt{x^2 + y^2 + [z - c(\sigma - i)]^2}.$$

Basic properties of potential W are as follows.

(1) Potential W includes the second, third, and partly fourth zonal harmonics of the Earth's attraction potential.

(2) The difference U–W includes terms whose order is equal to 10^{-9} and higher. This being the case, zonal harmonics, starting from the sixth one, have practically no difference, as well as tesseral and sector harmonics of this difference, with corresponding terms of the Earth's attraction potential.

(3) Potential W depends on three constants μ, c, and σ (or μ, J_2, J_3), which are currently determined with the best accuracy.

(4) The differential equations of spacecraft motion in a field with potential W are rigorously integrable by quadratures (in elliptic functions).

It should be noted that the influence of high harmonics is almost zero for a high-apogee spacecraft, which indicates the expediency of studying the evolution of the spacecraft orbit in a field with potential W.

The geopotential model EGM-96 [11] was used to calculate constants σ and c, while ephemeredes DE405/LE405 [12] were applied to construct the perturbing part of Hamiltonian caused by the Moon and the Sun. Thus, the real gravitational field U of the Earth can be represented as the sum of the intermediate gravitational field W and perturbing potential R_T, i.e., $U_E = W + R_T$.

The unperturbed generalized problem of two immobile centers has three independent integrals of motion, which are conserved over time. Existing perturbations from the Moon, Sun, neglected harmonics of the Earth's gravitational filed, solar light pressure, and other factors result in variations in these integrals over time.

In this case, we have the general problem of spacecraft motion under the action of a force with Hamiltonian $H(I, \theta; t) = H_0(I) + H_1(I, \theta; t)$, where H_0 represents the Hamiltonian of the problem's integrable part, and H_1 is an additional perturbation depending on position and velocity, and on time as well.

Since we want to obtain an orbit of SRT with maximum evolution, we should simply consider resonance orbits. These orbits can come into existence due to the action of the Moon and the Sun, as well as due to higher harmonics of the Earth's gravitational field.

The spacecraft motion in the unperturbed problem is integrated by quadratures, while in order to calculate perturbations from the Moon and the Sun, from higher harmonics of the Earth's gravitational field, and from solar light pressure with all necessary accuracy on long time intervals, the programs developed in ASC were used. They are based on algorithms presented in paper [13]. To calculate lunisolar perturbations, the algorithms described in [14] were used.

4.3. Third Stage: Numerical Calculations Made at Lavochkin NPO

Models and programs of numerical integration of the SRT equations of motion developed in ASC and Lavochkin NPO have been coordinated at the third stage. The final choice of the orbit was made in NPO with allowance for all constraints and taking into account perturbing factors as fully as possible. For every month of years starting from 2009 the conventional date of spacecraft launch was sought.

At first, for conventional dates of launch, those ranges of initial values of perigee argument and ascending node longitude were determined, for which the time of ballistic existence of spacecraft was no less than 9 years. Then, the functional Φ of the criterion of orbit efficiency was calculated for the chosen conventional dates of the launch.

Finally, for every month of 2011, calculations were made at Lavochkin NPO on the daily choice of the initial parameters of an orbit that provides the maximum criterion Φ, which characterizes the observation of the celestial sphere, preset time of ballistic existence of the spacecraft, and duration of shadowing.

In order to make provisions for synthesis pf high-quality images with the help of a ground–space interferometer, one needs to have an orbit whose parameters would be strongly variable under the action of gravitational perturbations from the Moon and the Sun. Simultaneously, these orbits ensure a high resolution of the interferometer and high quality of images, since a projection of the Earth–spacecraft vector onto the picture plane runs through all values both in magnitude (from zero to several hundred thousand kilometers) and in position angle.

The synthesis of the best orbit was considered in the formulation of searching for an extreme value of quality functional $\Phi^{opt} = \mathrm{extr}\boldsymbol{\Phi}\,(\mathbf{X}, \mathbf{Y})$, $\mathbf{X} \in D$, $\mathbf{Y} \in S$, and $\mathbf{X} = \{x_1, \ldots, x_6\}$ are initial elements of the spacecraft orbit, control variables. The admissible region D of variation of control variables is determined by the system of equalities and constraints $F_i(\mathbf{X})\ \{\le, =, \ge\}\ b_i$, $i = 1, 2, \ldots, n$. This system corresponds to requirements to spacecraft orbit and constraints on it imposed by the carrier launcher, booster block, time of active existence, power-mass characteristics of the spacecraft, and regimes of functioning of the spacecraft's onboard systems and of the ground-based segment of control.

When choosing the initial parameters of the working orbit of *Spektr-R*, one must take into account the following constraints: the space experiment duration is no less than 5 years; the time of ballistic existence is no less than 9 years; the interval of shadowing per orbit should not exceed 2 h (taking into account possible correction of phasing; the height of the working orbit apogee is 300 000–360 000 km; the time of ballistic existence and duration of shadowing per orbit should be kept with allowance made for the scatter of the initial parameters of the working orbit during the launch (on the level of 3σ, it is equal to about 8 h in the period of revolution). In this case, the total increment of characteristic velocity due to all corrections should not exceed 60 m/s.

The conditions of making observations correspond to a discrete set S of parameters that determine structure of the quality function. When selecting the initial parameters of the working orbit, it is necessary to maximize the value of Φ, which is a criterion of orbit quality.

Parameters of the working orbit can be conventionally divided in two groups, i.e., constants and control variables. The constants are as follows: inclination of the orbit determined by latitude of the launch point (Baikonur spaceport), which is assumed to be 51.4°, and the initial height of pericenter, which is determined by the power capabilities of a booster block and is taken to be equal to 400 km.

The argument of the pericenter and longitude of the ascending node are selected as control variables. The height of apocenter varies within the limits of 300 000–360 000 km, and it is determined by power capabilities of the booster block.

Thus, one must choose initial elements of the spacecraft orbit that maximize the criterion of orbit quality taking into account all imposed constraints. For the sake of definiteness, the range of conventional dates of the launch was taken as November 30 through December 3, 2009.

At first for the considered conventional dates of launch those ranges of initial values of the perigee argument and longitude of ascending node were determined at which the time of ballistic existence would be no less than 9 years. The results of these calculations were tabulated. An example of calculations for November 30, 2009 is presented in Table 1.

For every combination of initial values of the perigee argument and longitude of the ascending node, this table presents the times of ballistic existence in days. The combinations of initial parameters for which the time of ballistic existence is no less than 9 years are shaded.

To simplify the process of preparing spacecraft onboard systems for launch, we have selected those values of perigee argument at which the same value of the perigee argument is found for all considered conventional dates of launch. After this, of the obtained values of perigee argument that was chosen at which the condition of maximum duration of residence in the Earth's shadow (≤2 h) was satisfied and the maximum value of quality criterion functional was reached. In the considered range of conventional dates of launch the initial value of the perigee argument is assumed to equal 292°.

Variations in the control variables in the range of their admissible value and search for the maximum orbit quality criterion have shown that, for every conventional launch date, there is an optimal value of longitude of the orbit ascending node. These values are presented in Table 2.

Taking into account the optimization of the requirements and constraints listed above, as well as capabili-

Table 1

Ω																					
320	9	9	9	9	9	9	9	1	1	9	1	5	9	9	4	1	1	1	9	1	1
321	8	1	9	9	9	9	9	8	5	9	9	9	1	3	6	9	9	1	1	1	0
322	1	1	9	9	3	9	9	5	1	9	1	1	7	8	8	4	1	1	2	1	2
323	4	9	9	9	9	9	6	9	9	5	5	8	6	1	1	1	1	1	1	7	3
324	2	9	9	9	9	6	6	6	1	9	9	9	7	1	1	1	9	7	9	9	1
325	7	9	1	9	6	1	2	9	9	1	1	1	1	1	3	1	9	9	9	9	1
326	9	2	6	6	**1**	9	6	1	9	1	1	1	1	1	9	8	4	1	1	1	1
327	7	9	6	2	9	6	6	9	9	6	1	1	7	1	9	7	1	1	1	1	1
328	6	9	2	9	6	6	6	6	1	6	1	7	1	1	1	9	1	1	1	1	1
329	9	7	2	9	9	1	1	1	1	1	9	3	1	1	9	1	1	1	1	1	1
330	7	6	9	9	2	2	1	2	1	6	9	3	1	9	1	1	1	1	1	1	1
331	7	9	9	2	6	2	2	2	1	6	7	3	1	1	1	1	1	1	1	1	1
332	9	7	9	9	2	4	6	1	4	6	7	9	9	3	1	1	1	1	1	1	1
333	7	9	7	9	4	4	6	2	2	6	6	6	6	1	1	1	1	1	1	1	1
334	9	7	6	7	7	4	2	2	2	6	4	2	1	1	1	1	1	1	1	1	1
335	5	7	9	7	7	4	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1
336	2	2	2	2	7	4	2	2	2	2	1	2	1	1	1	1	1	1	1	1	1
337	2	2	2	2	7	4	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
338	2	2	2	7	7	7	2	2	4	2	2	2	2	2	2	1	1	1	1	1	1
339	2	2	2	9	9	2	2	7	2	2	2	2	2	2	1	1	1	1	1	1	1
340	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1
ω	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310

Table 2

Date of launch	Nov. 30, 2009	Dec. 1, 2009	Dec. 2, 2009	Dec. 3, 2009
Longitude of ascending node	331	306	354	344
Functional of orbit quality	19940	19337	19454	19659

ties of the carrier launcher, we have chosen the following initial parameters of the spacecraft working orbit:

Pericenter height, km	400.0
Apocenter height, km	330000.0
Inclination, deg	51.4
Perigee argument, deg	292.0
Longitude of ascending node, deg	(Table 2)
Period of revolution, h	205.0

When calculating the functional, its components were estimated for different projections of the base onto (uv) plane. Seven intervals $\mathbf{I}(i)$ of variations in this base projection onto the (uv) plane were considered, i.e., 0–5000 km, 5000–15000 km, 15000–50000, 50000–150000, 150000–200000, 200000–250000, and 250000–450000 km.

The functional was calculated In the same manner for July 18, 2011. As a result of the optimization, the following initial parameters of the working orbit were

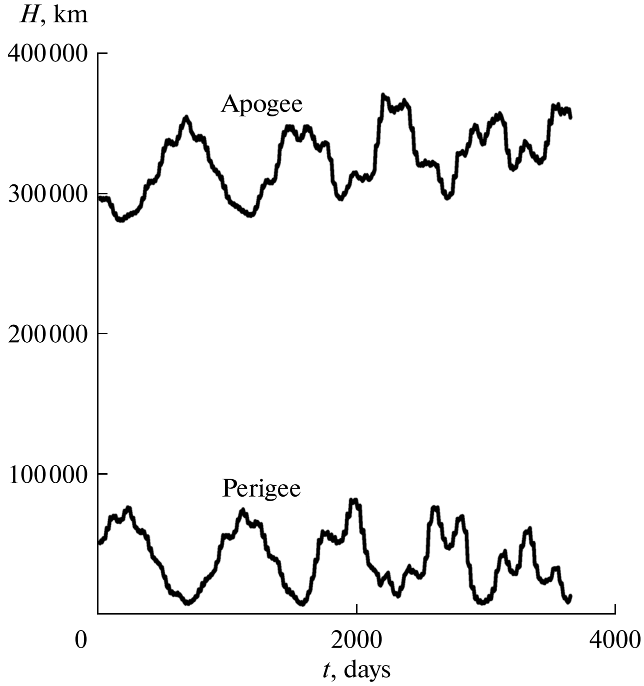

Fig. 2. Evolution of perigee height H_π and apogee height H_a of the orbit.

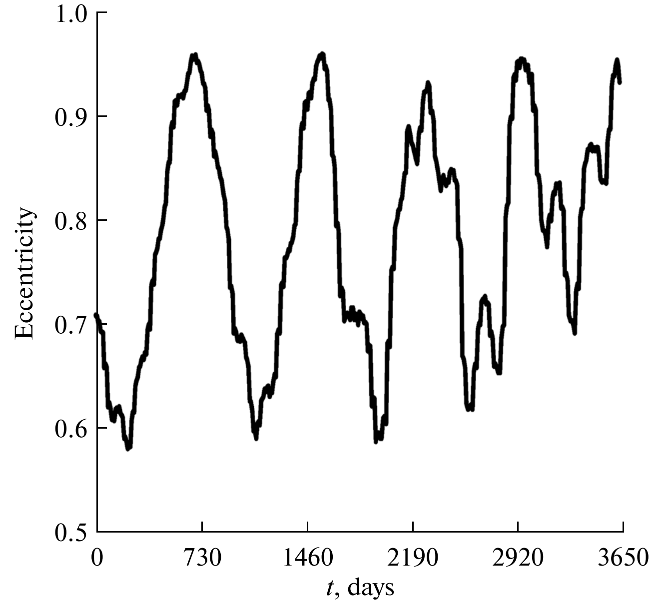

Fig. 3. Evolution of eccentricity of SRT orbit.

selected: $H_a = 330000$ km; $H_\pi = 600$ km; $\omega = 302°$; $i = 51.57°$; $\Omega = 342.2°$; $t_0 = 05.31.18$ (LCT). In this case, the orbit lifetime is no less than 9.6 years, and quality criterion $\Phi = 16208$.

Under the action of gravitational perturbations from the Moon and the Sun, and due to the noncentrality of the Earth's gravitational field the spacecraft working orbit parameters are changed substantially.

Figures 2–4 present the plots of evolution of basic parameters of a nominal orbit over nine years since the moment of orbit correction in May 2012. Figure 5 shows evolution of the direction of the vector of normal to the orbit plane onto the celestial sphere. As is seen in these figures the evolution of elements of the working orbit has a periodic character with a period of about 900 days.

Nominal initial values of orbit parameters after injection for the launching date July 18, 2011 were taken as the initial conditions. The time of spacecraft lifetime in days (from the moment of separation of the spacecraft from the *Fregat* booster block) are laid off in the plots as abscissa. The real orbit elements at the moment of separation are presented in Table 3.

Since such (strongly evolving) orbits are highly dependent on initial data, a small shortage of the perigee height (Table 3) has resulted in the possibility for the spacecraft on a realized orbit to cease to exist at the end of 2013. In order to avoid this, a correction of the orbit was made on March 1, 2012.

CONCLUSIONS

In this paper, we have demonstrated the theoretical possibility of practical realization of a high-apogee and strongly evolving orbit for the RadioAstron project using the technical means stipulated by the project.

Methods have been developed to estimate the orbit quality for ground–space radio interferometers. In order to search for orbits with the required properties, algorithms and programs of directional selection of orbit parameters are developed based on these estimates. The orbits, which ensure good conditions for observing sources located in the neighborhood of the north and south poles of the Galaxy are constructed.

Table 3

Quantity	Value
Date and time (LCT) of launch	July 18, 2011 05.31.18
Date and time (LCT) of separation	July 18, 2011 09.06.58,6
Apocenter height, km	333455
Pericenter height, km	578
Perigee ergument, deg	302.02
Orbit inclination, deg	51.57
Longitude of ascending node, deg	342.2
Period of revolution, days	8.32

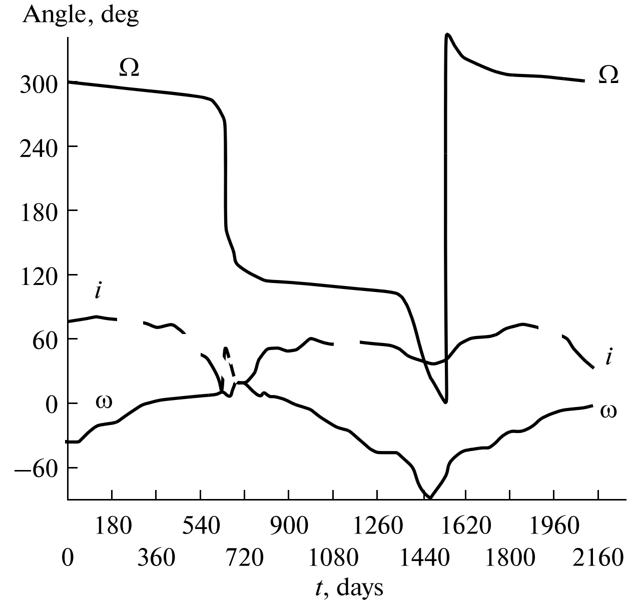

Fig. 4. Evolution of angular parameters.

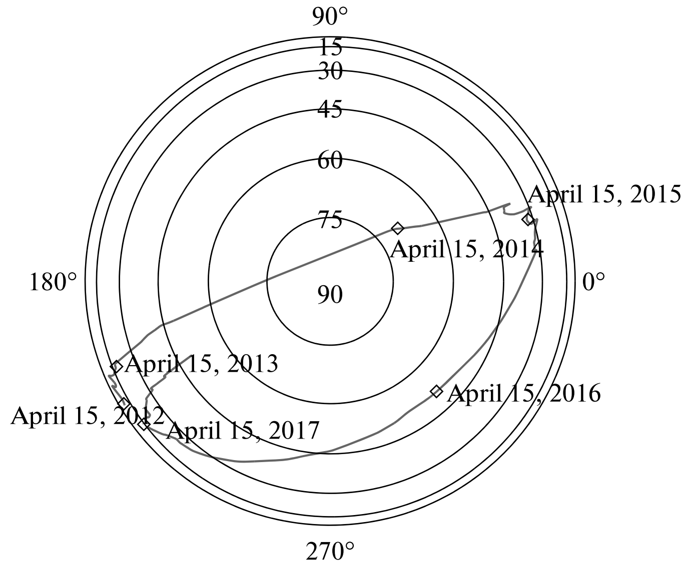

Fig. 5. Evolution of the projection of direction of a normal vector to the orbit plane onto the celestial sphere.

Work has been done to design such an orbit for the RadioAstron project with an apogee radius of up to 360000 km. Algorithms and programs have been elaborated for constructing periodic solutions of the restricted three-body problem for the planar and three-dimensional cases. Using this software, various families of periodic solutions have been constructed. It has been shown that even orbits with apogees of less than 300000 km (so that they formally are not in the sphere of action of the Moon) can demonstrate intense motion of the apsidal line. Orbits with a period of 9 days are chosen as promising for use in the RadioAstron project.

For example, the technical characteristics of carrier launcher *Zenith* and booster block *Fregat-SB* allowed one to launch the SRT into the orbit with an initial apogee of 333455 km. Studying the dependence of orbit evolution on the date of launch, we have shown that the main contribution to orbit evolution is made by the Moon and, for every month, one can find the starting moment that ensures the optimal orbit evolution in order to implement the scientific program. Technical capabilities of a low-thrust propulsion system can provide for necessary orbit correction to support the required evolution taking into account real measurements of orbit parameters in the course of the mission. It is shown that, in order to cut the duration of shadowing, it is sufficient to use a small impulse of the CPS (correction propulsion system). In the future, it is planned to make an explicit simulation of CPS activation for these purposes.

Thus, the technical capabilities of the project allow one to realize a high-apogee orbit (with strong evolution of osculating elements) and to ensure the solution of scientific research tasks of the mission.

When designing the orbit, no allowance was made for solar light pressure or the force of unloading angular momentum of flywheels of the stabilization system, since they depend significantly on the schedule of observations. Therefore, approximately once in a year or half-year, one must correct the spacecraft orbit.

The *Spektr-R* spacecraft was successfully launched on July 18, 2011, and its functioning in the working orbit for more than 2 years has shown the adequacy of the developed method of choosing initial parameters of the working orbit and the correctness of its particular realization.

ACKNOWLEDGMENTS

The authors thank the following persons from the Keldysh Institute of Applied Mathematics for numerous fruitful discussions on issues of choosing orbits for a space radio telescope that continued for two decades, including V.V. Beletsky, M.A. Vashkov'yak, V.A. Egorov†, and V.V. Sazonov. We also thank collaborators from the Shternberg Astronomical Institute N.V. Emel'yanov (for useful discussions on Euler orbit) and V.V. Chazov who checked computations within the scope of this problem taking into account solar light pressure. The RadioAstron project is implemented by the Astro-Space Center of Lebedev Physical Institute and by the Lavochkin NPO under a contract with the Russian Space Agency and together with

† Deceased.

many scientific and engineering institutions in Russia and other countries.

REFERENCES

1. Sedov, L.I., Moonward orbits of spacecraft, *Iskusstvennye Sputniki Zemli,* 1960, no. 5, p. 3.
2. Lidov, M.L., Evolution of orbits of planetary artificial satellites under the action of gravitational perturbations of external bodies, *Iskusstvennye Sputniki Zemli*, 1961, no. 8, p. 5.
3. Vashkov'yak M.A. and Lidov M.L., On evolution of some types of orbits of the Earth's satellites, *Kosm. Issled.*, 1990, vol. 28, no. 6, pp. 803–807.
4. Kreisman, B.B., Estimation of orbits for a groud-space radio interferometer, *Preprint of Lebedev Physical Inst., Russ. Acad. Sci.,* Moscow, 1996, no. 61.
5. Kreisman, B.B., Symmetrical periodic solutions in planar restricted three-body problem, *Preprint of Lebedev Physical Inst., Russ. Acad. Sci.*, Moscow, 1997, no. 66.
6. Kardashev, N.S., Kreisman, B.B., and Ponomarev, Yu.N., The new orbit and new capabilities of the RadioAstron project, in *RadioAstronomicheskaya tekhnika i metody* (Radio Astronomy Facilities and Methods), Moscow: Trudy FIAN, 2000. vol. 228, pp. 3–12.
7. Marsh, J.G., et al., The GEM-T2 gravitational model, *J. Geophys. Res.: Solid Earth*, 1990, vol. 95, no. B13, pp. 22043–22071.
8. Standish, E.M., et al., JPL planetary and lunar ephemerides DE403/LE403, *JPL Inter Office Memorandum*, 1995, no. 314, pp. 10–124.
9. Aksenov, E.P., Grebennikov, E.A., and Demin, V.G., General solution of the problem of satellite motion in the regular field of the Earth's attraction, *Iskusstvennye Sputniki Zemli*, 1961, no. 8, p. 64.
10. Aksenov, E.P., *Teoriya dvizheniya iskusstvennykh sputnikov Zemli* (The Theory of Motion of the Earth's Artificial Satellites), Moscow: Nauka, 1977.
11. Lemoine, F.G., Kenyon, S.C., Factor, J.K., et al., The Development of the Joint NASA GSFC and National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96. /NASA/TP-1998-206861. 1998. Goddard Space Flight Center, Greenbelt, Maryland. http://www.nima.mil/GandG/wgs-84/egm96.html
12. Standish, E.M., Newhall, X.X., Williams, J.G., and Folkner, W.F., JPL planetary and lunar ephemeris DE405/LE405, *JPL Inter Office Memorandum*, 1998, no. 312.
13. Aksenov, E.P., Emel'yanov, N.V., and Tamarov, V.A., Practical application of intermediate orbit of a satellite: Formulas, programs, and tests, *Trudy GAISh MGU*, 1988, vol. 59, pp. 3–40.
14. Aksenov, E.P. and Chazov, V.V., *Model' dvizheniya ISZ. Glavnaya problema. Osnovnye algoritmy* (Model of Satellite Motion: Fundamental Problem and Basic Algorithms), Moscow: MGU, GAISh, AI RAN, 2011.

Translated by A. Lidvansky